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Numerical model based on SPH method to simulate Friction Stir Welding

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Abstract
In the present work, a numerical model, based on the smoothed particle hydrodynamics method (SPH), is developed to simulate Friction Stir Welding process (FSW). This model considers a non Newtonian fluid near the tool region using a thermo-mechanical constitutive law. We limit ourselves to bi-dimensional problems. A comparison with an industrial CFD code is performed.

Résumé
Dans le présent travail, un modèle numérique, basé sur la méthode sans maillage SPH, est développé pour simuler le soudage par friction et malaxage (FSW). Ce modèle numérique considère, au voisinage de l'outil, que le matériau se comporte comme un fluide non newtonien avec une loi de comportement thermo-mécanique. Nous nous limitons ici à des problèmes bidimensionnels. Une comparaison avec un code industriel de mécanique des fluides CFD est présentée pour valider notre modèle.

1. INTRODUCTION

Friction Stir Welding (FSW) was invented by the British Welding Institute TWI since 1990s for aluminum sheets. Thomas et al. [1] have shown that the main advantage of this technique is its ability to joining in solid state a class of metal alloys which is generally difficult to weld by conventional welding processes. Joining two workpieces by FSW consists in heat generation due mainly to the shoulder and material mixing thanks to the pin.

Numerical modelling of FSW has been investigated by several authors considering thermal or thermo-mechanical framework [2-8]. Material mixing near the welding tool is very difficult to simulate because this involves very large deformations. Different formulations have been proposed in Eulerian, Lagrangian or Arbitrary Lagrangian Eulerian frameworks. Eulerian formulation is appropriate to describe material flow for large deformations thanks to fixed mesh grid but this formulation cannot be used for problems involving free surfaces. Lagrangian formulation is well adapted for history dependence in solids and for simulation of material flow with free surfaces but mesh distortions requires special procedure as remeshing. The ALE formulation has been proposed to benefit from both advantages of Eulerian and Lagrangian formulations [3]. In this technique, one has to describe the motion of the mesh and material particles separately with respect to a reference domain. ALE has been applied successfully in many fields and particularly in metal forming processes. Despite the intensive development of these techniques, material mixing in FSW process remains very difficult to achieve numerically and especially when using finite element method [2, 3]. An alternative solution for the simulation of this process is the use of meshless methods. To our knowledge, few numerical contributions using meshless techniques are available. Alfaro et al. [9] have proposed an algorithm based on natural element methods (NEM) to simulate the mixing zone in 2D.

In the present work, we propose a meshless algorithm based on the smoothed particle hydrodynamics (SPH) [10] [12-15] to simulate FSW process. We consider the welding zone as a non newtonian fluid in which viscosity is temperature dependent. The mixing region is also treated as a weakly compressible fluid. This simplification allows us to use a state equation to determine the fluid pressure. However, compressibility is adjusted to reduce the sound speed to obtain a reasonable time step in the model. In this work, the contact between the tool and the material particles is managed by the forces due to the pressure term; this allows us to avoid penetration of particles within the material [16]. We limit ourselves to 2D simulations but 3D computations will be presented in a further paper to show material mixing in the thickness direction.

The layout of this paper is as follows. In section 2, we present the main equations required for the thermo-mechanical problem to be solved. Heat generation is mainly due to the high strain gradient in the neighborhood of the welding tool. Section 3 gives the principle of SPH method and time discretization of the different equations. In section 4, we present some numerical results showing the performance of the proposed algorithm. Results of the proposed model are compared with those obtained using the industrial code Fluent. This latter is based on computational fluid dynamics and uses Eulerian formulation.

2. GOVERNING EQUATIONS

Friction stir welding is a complex thermo–mechanical process that requires accurate knowledge of the relations existing between the main parameters of the process such as plunge depth, travel speed, rotation speed, thermo-mechanical properties, tool geometry, etc. In many contributions, the mixing zone is considered as a high viscosity incompressible fluid and the flow is obtained using computational fluid dynamics. In some other cases, the deformation is modeled using solid mechanics and numerical methods to solve the resulting nonlinear problem to compute the different variables as velocity, pressure, temperature… In the present work, we consider a thermo-mechanical coupling with

a constitutive law that depends only on temperature. The resulting problem is then described by the conservation laws including:
Mass conservation:

$$\frac{d\rho}{dt} = -\rho\nabla.v \quad (1)$$

Momentum conservation:

$$\rho\frac{dv}{dt} = \nabla\sigma \quad (2)$$

Energy conservation:

$$c_p\frac{dT}{dt} = \nabla.(k\nabla T) + q_v \quad (3)$$

Heat source:

$$q_v = -\beta \;\; \tau : \nabla v \quad (4)$$

where v is the velocity vector, ρ the specific density, σ is the Cauchy stress tensor, c_p is the specific heat capacity at constant pressure, T denotes the temperature field, k denotes the thermal conductivity of materials and β represents the fraction of mechanical energy transformed to heat assumed to be 0.9 [11]. The stress tensor can be written in the following form:

$$\sigma = -pI + \tau \quad (5)$$

where p is the hydrostatic pressure, I is the identity tensor and τ is the deviatoric stress tensor given by:

$$\tau = \mu\,[{}^t\nabla v + \nabla v - \frac{2}{3}(\nabla.v)\,I] \quad (6)$$

where μ is the viscosity of the fluid considered in this study as temperature dependent and is given in the following form:

$$\mu = A\left(1 - \left(\frac{T - T_{ref}}{T_{melt} - T_{ref}}\right)^m\right) \quad (7)$$

where A and m are material parameters, T_{ref} is a reference temperature and T_{melt} is the melting temperature. The pressure p is obtained using the following state equation:

$$P = c^2\rho \quad (8)$$

where c is the sound speed [12].

3. NUMERICAL MODEL

The proposed numerical model is based on a semi implicit time discretization scheme and a space discretization using Smoothed Particle Hydrodynamics SPH approach. This technique belongs to meshless methods and particularly adapted to lagrangian framework. The main features of SPH approach, which is based on integral interpolant, are described in detail in the following works (Monaghan, 1989) [13], (Monaghan, 1992) [14], (Monaghan, 2005) [15], (Liu, 2003) [12], [Benz, 1990] [17]. Here, we will only refer to the representation of the constitutive equations in SPH notation.

3.1 PRINCIPLE OF SPH APPROACH

In SPH approach, the main idea is to approximate any arbitrary function $f(r)$ by:

$$f(r) = \int_D f(r')W(r - r', h)\, dr' \quad (9)$$

where h is called the smoothing length and W is the weighting function with a compact support h. The approximation of equation (9) at an interpolated point of coordinate r, in discrete notation, leads to the following approximation:

$$f(r) = \sum_{b=1}^{N_p} f(r_b) W(\|r - r_b\|, h) V_b \tag{10}$$

where N_p is the number of neighboring points and V_b is the volume associated with point b. In the case where the interpolated point is a particle, the volume V_b is replaced by the ratio of its mass m_b and mass density ρ_b. The spatial derivatives of a function $f(r)$ can be calculated through exact differentiation of the weighting function W according to the equation:

$$\nabla_r f(r) = \sum_{b=1}^{N_p} (f(r) - f(r_b)) \nabla_r W(\|r - r_b\|, h) V_b \tag{11}$$

In this work, we use the following weighting function of cubic spline type [12]:

$$W(\|r - r_b\|, h) = \alpha_d \begin{cases} 1 - \frac{3}{2} z^2 + \frac{3}{4} z^3 & if \quad z \leq 1 \\ \frac{1}{4}(2 - z)^3 & if \quad 1 < z \leq 2 \\ 0 & if \ z > 2 \end{cases} \tag{12}$$

where z and α_d are given by:

$$\begin{cases} z = \frac{\|r - r_b\|}{h} \\ \alpha_d = 10/7\pi h^2 \quad in\ 2D \end{cases} \tag{13}$$

3.2 SPACE DISCRETIZATION

The domain under study is represented by a set of distributed particles. No connectivity for these particles is needed. So, the integral representation method is used for field function approximation. The kernel approximation is then further approximated using particles. It is done by replacing the integration in the integral representation of the field function and its derivatives with summations over all the corresponding values at the neighboring particles in a local domain called the support domain.
Using SPH approximation technique, the governing equations (1)-(8) are discretized resulting in a set of ordinary differential equations:

$$\frac{d\rho_a}{dt} = -\sum_{b=1}^{Np} m_b (v_a - v_b) \nabla_{r_a} W \tag{13}$$

$$\frac{dv_a}{dt} = -\sum_{b=1}^{Np} m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \nabla_{r_a} W + \sum_{b=1}^{Np} \frac{m_b}{\rho_a \rho_b} \frac{4\mu_a \mu_b}{(\mu_a + \mu_b)} \frac{(v_a - v_b).(r_a - r_b)}{(r_a - r_b)^2} \nabla_{r_a} W \tag{14}$$

$$C_{pa} \frac{dT_a}{dt} = \sum_{b=1}^{Np} \frac{m_b}{\rho_a \rho_b} \frac{4k_a k_b}{(k_a + k_b)} \frac{(T_a - T_b)(r_a - r_b)}{(r_a - r_b)^2} \nabla_{r_a} W + \mathrm{q_v} - \mathrm{h}(\mathrm{T_a} - \mathrm{T_{ref}}) \tag{15}$$

$$q_v = -\beta \sum_b \frac{2 m_b \mu_a \mu_b}{\rho_a \rho_b (\mu_a + \mu_b)} \frac{[(v_a - v_b).(r_a - r_b)]^2 (r_a - r_b)}{(r_a - r_b)^2} \nabla_a W_{ab} \tag{16}$$

$$\mu_a = A \left(1 - \left(\frac{T_a - T_{ref}}{T_{melt} - T_{ref}} \right)^m \right) \tag{17}$$

$$p_a = c^2 \rho_a \tag{18}$$

To avoid penetration of the particles representing weld material into the tool, boundary particles are forced to satisfy the same equations as fluid particles. These particles move together with the following velocities:

$$v_i = \omega r_{i,c} + v_t, \qquad r_{i,c} = (x_i - x_c, y_i - y_c, 0) \tag{19}$$

where ω is the rotational velocity and v_t is the translational velocity of the center of the tool, x_c and y_c are coordinates of the tool's center.
Particles are moved using the XSPH variant (Monaghan, 1989) [13]:

$$\frac{dr_a}{dt} = v_a + \varepsilon \sum_b \frac{m_b}{\bar{\rho}_{ab}} (v_a - v_b) W_{ab} \tag{20}$$

where $\bar{\rho}_{\mathrm{ab}} = (\rho_a + \rho_b)/2$ and ε is a given parameter depending on the problem to be solved. This method moves the particle a with a velocity that is close to the average velocity in its neighborhood.

3.3 TIME INTEGRATION

To solve numerically the problem (13-18), we use the time scheme proposed by Allen and Tildesley [18] based on explicit velocity Verlet algorithm.
Using this algorithm, the new positions, densities, velocities and temperatures in the welded materials are found by time integration of equations (13-15) as follows:

$$\begin{cases} \rho_a^{n+1} = \rho_a^n + 0.5\Delta t (\dot{\rho}_a^n + \dot{\rho}_{\mathrm{a}}^{n+1}) \\ v_a^{n+1} = v_a^n + 0.5\Delta t (\dot{v}_a^n + \dot{v}_{\mathrm{a}}^{n+1}) \\ T_a^{n+1} = T_a^{n-1} + 0.5\Delta t (\dot{T}_a^n + \dot{T}_a^{n+1}) \\ r_a^{n+1} = r_a^n + \Delta t\, v_a^n + 0.5\Delta t^2\, \dot{v}_a^n \end{cases} \tag{21}$$

where $\dot{v}_a = \frac{dv_a}{dt}$, $\dot{\rho}_a = \frac{d\rho_a}{dt}$, $\dot{T}_a = \frac{dT_a}{dt}$

4. NUMERICAL DISCUSSION

In the present study, the workpiece is an aluminum alloy plate with a dimension of 50×20 mm. The workpiece material is assumed to be incompressible with variable viscosity μ which is given by equation (17) with the following parameters:

$A = 200$ Mpa.s^{-1}, $m = 1$, $T_{ref} = 25$ °C, $T_{melt} = 502$ °C.

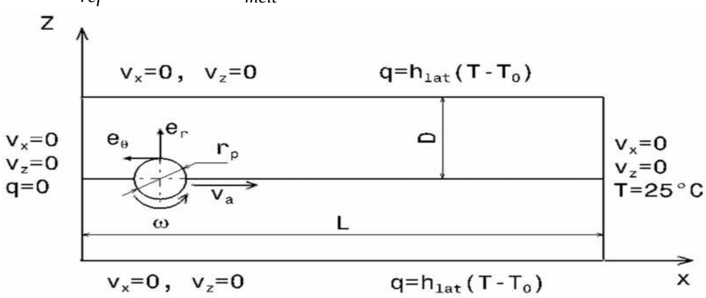

Figure 1: Geometry and boundary conditions of FSW configuration treated by SPH

The tool is considered as rigid. The tool pin is 2.5 mm in radius ($r_p = 2.5\,\mathrm{mm}$) and the thermo-mechanical material properties of the plate are as follows:

$\rho(kg/m^3)$	$C_p(J/K.kg)$	$k(W/m.K)$
2780	875	140

Table 1: Physical proprieties

In the present work, the welding velocity v_a and the angular velocity ω of the tool have the following values:

$$v_a = 2mm/s \qquad \omega = 20rad/s$$

The material flow around the tool pin is reported in the figure 2 for different computation time. As it can be shown from these results, the SPH method is very useful to simulate the extreme conditions of the thermo-mechanical material flow around the tool. The distribution of the temperature field is also reported in figure 3.

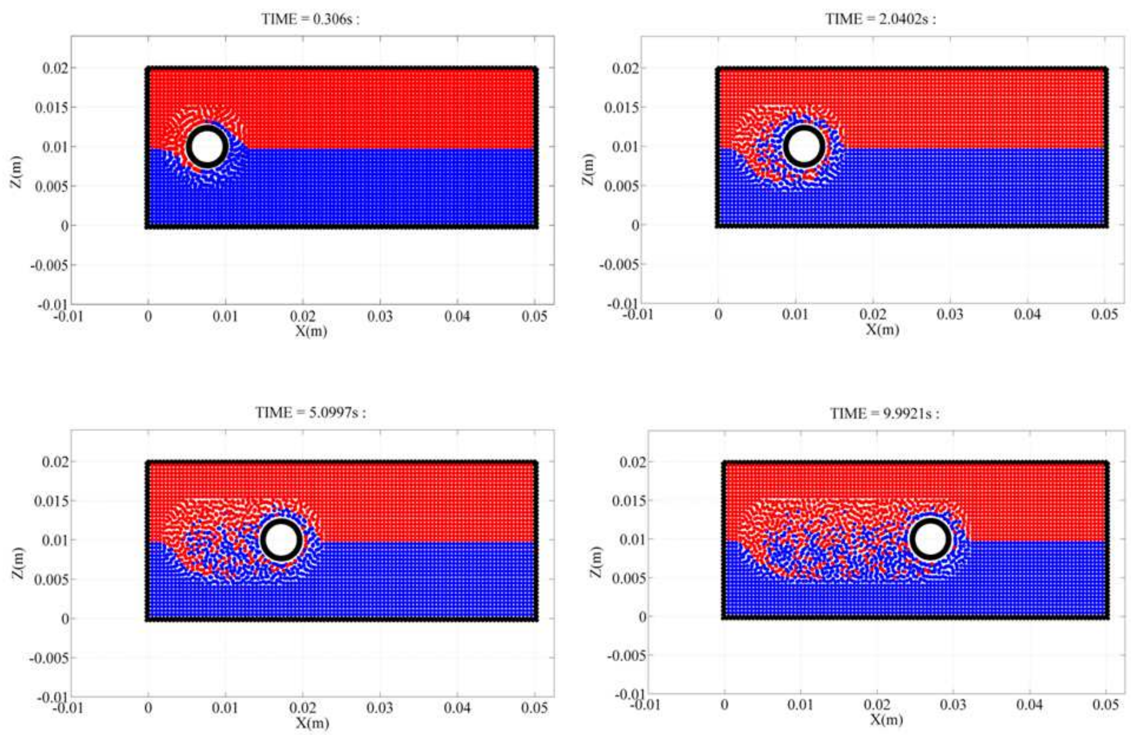

Figure 2: Thermo-mechanical material flow around the tool pin

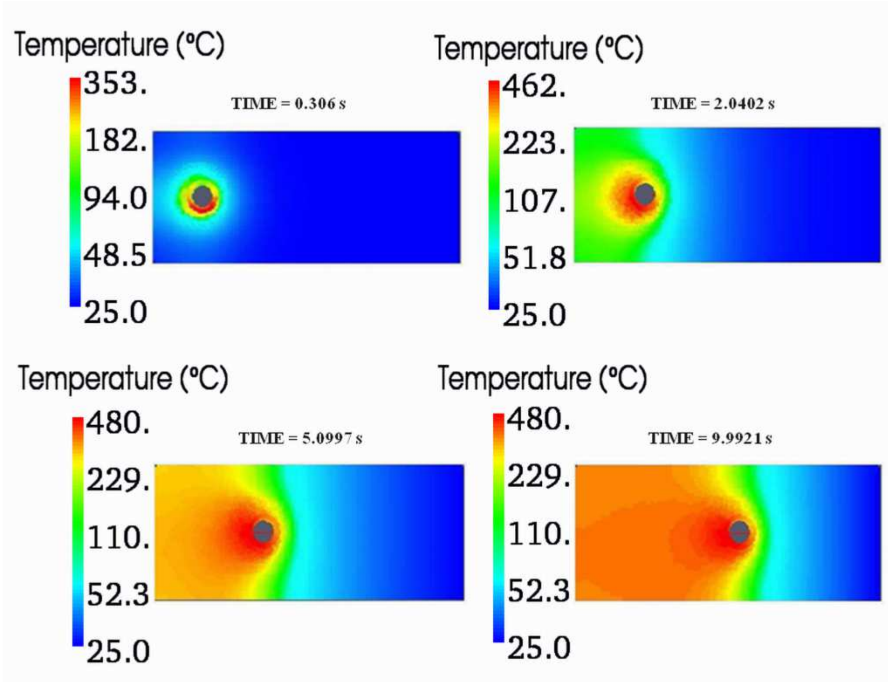

Figure 3: Temperature distributions corresponding to the material flow of figure **2**

In order to validate the numerical results of the SPH approach, we propose to perform the same computation by using Fluent software. This code is based on an Eulerian formulation and a finite volume discretization. Thus, two equivalent configurations between the Lagrangian and Eulerian formulations have been considered (see figures 1 and 4). The SPH calculation is performed until the tool reaches the plate center. This tool position corresponds to the time t = 9s. In the Eulerian formulation, unsteady calculation with the same time t is also performed. The two calculations use the same constitutive law (equation 17). Figures 7 and 8 show the comparison between SPH and Fluent for the temperature evolutions along the horizontal and vertical cut views (see figure 6). It can be observed that a relative error of at least 5% is obtained confirming the relevance of the proposed algorithm.

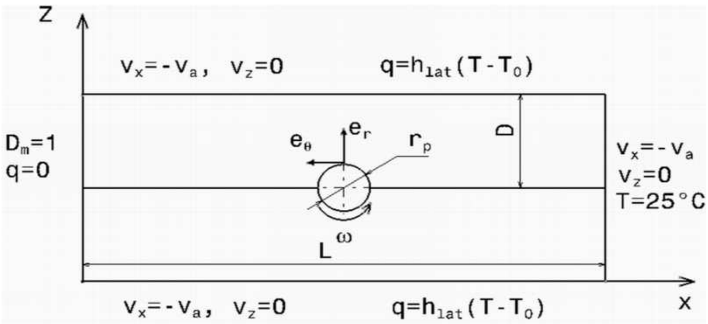

Figure 4: Geometry and boundary conditions of FSW configuration treated by Fluent

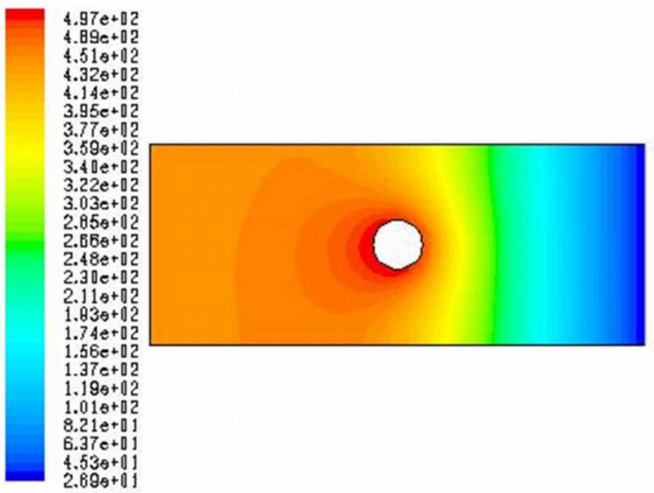

Figure 5: Temperature distribution by Fluent

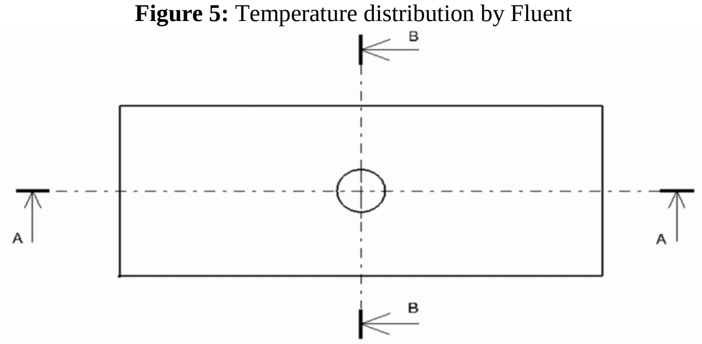

Figure 6: Horizontal cut view (A-A) and vertical cut view (B-B)

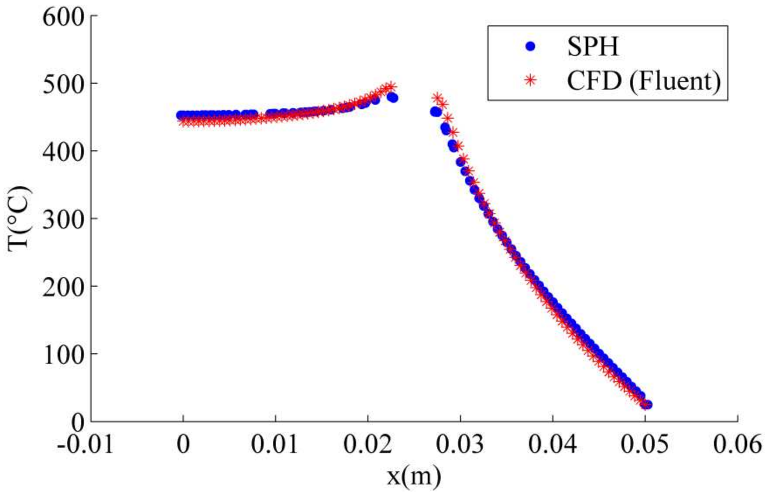

Figure 7: Temperature evolution along the horizontal cut view (A-A)

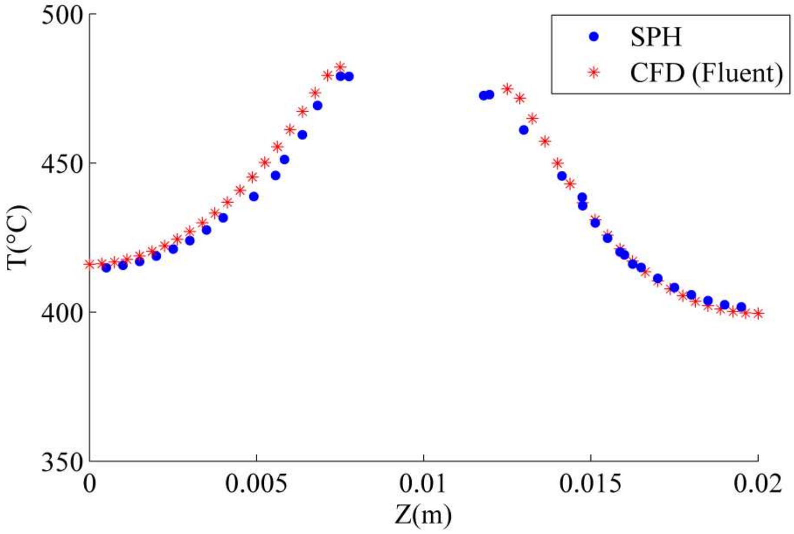

Figure 8: Temperature evolution along the vertical cut view (B-B)

5. CONCLUSION

In this contribution, we have proposed a first model based on based the smoothed particle hydrodynamics method to model Friction Stir Welding process. The main advantage of this technique concerns the simulation of the thermo-mechanical material flow around the tool which is very difficult to obtain with other numerical methods such as finite element method.
In a next work, we will propose an extension of the present model to the three dimensional case with a complex tool geometry. A more realistic constitutive law dependent on temperature, strain rate and work hardening will be also studied.

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