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Numerical Study of Natural Convection and Surface Radiation in a Square Cavity submitted to Cross Gradients of Temperature

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Abstract

The present paper reports numerical results of natural convection and surface radiation within a square cavity filled with air and submitted to discrete heating and cooling from all its walls. Two heating modes are considered: in the first mode, named *VB* heating mode, the cavity is heated discreetly on the left vertical and bottom horizontal walls, while in the second mode, called *VT* heating mode, the left vertical and top horizontal walls of the cavity are heated. The remaining portions of the walls are considered adiabatic. The parameters governing the problem are the emissivity of the walls ($0 \le \varepsilon \le 1$), the relative lengths of the active elements ($B = h'/H' = \ell'/L' = 0.5$) and the Rayleigh number ($10^4 \le Ra \le 10^7$). The effect of these parameters on heat transfer and fluid flow within the cavity is examined. Results of the study are presented and compared to the case of pure natural convection for various typical combinations of the governing parameters in terms of flow and temperature patterns, average convective, radiative and total Nusselt numbers. A comparison between results of the two heating modes is also conducted.

Keywords: Natural convection, thermal radiation, discrete heating, heat transfer, cross gradients of temperature, numerical study.

Résumé

Le présent travail présente les résultats numériques de la convection naturelle couplée au rayonnement thermique des surfaces dans une cavité carrée remplie d'air et chauffée et refroidie discrètement à travers ses quatre parois. Deux modes de chauffage sont considérés : dans le premier mode de chauffage nommé *VB*, la cavité est chauffée par le côté vertical gauche et par le bas, alors que le deuxième mode de chauffage baptisé *VT*, la cavité est chauffée discrètement par la paroi verticale gauche et par le haut. L'effet des paramètres régissant le problème sur le transfert de la chaleur et l'écoulement du fluide dans la cavité est examiné. Les résultats de cette étude sont présentés et comparés au cas de la convection naturelle pure pour diverses combinaisons typiques de ces paramètres.

Mots clés : Convection naturelle, Rayonnement thermique, Transfert de chaleur, Gradients croisés de température, Etude numérique.

Nomenclature

A_r	aspect ratio of the cavity (A_r = L'/H')
B	relative lengths of the active portions, $B = h'/H' = \ell'/L'$
cv	convection
F_{ij}	view factor from S_i surface to S_j one
g	acceleration due to gravity, m/s^2
h'	height of the vertical active portions, m
H'	height of the cavity, m
J_i	dimensionless radiosity, $J'_i/\sigma T'^4_C$
ℓ'	width of the horizontal active elements, m
L'	length of the cavity, m
N_r	convection-radiation interaction parameter, $N_r = \sigma T'^4_C H'/\lambda(T'_H - T'_C)$
Nu	average Nusselt number
Pr	Prandtl number, $Pr = \nu/\alpha$
Q_r	dimensionless radiative heat flux, $Q_r = Q'_r/\sigma T'^4_C$
Ra	Rayleigh number, $Ra = g\beta(T'_H - T'_C)H'^3/\alpha\nu$
rd	radiation
t	dimensionless time, $t = t'\alpha/H'^2$
T'	dimensional fluid temperature, K
T	dimensionless fluid temperature, $T = (T' - T'_C)/(T'_H - T'_C)$
T'_C	temperature of the cold elements, K
T'_H	temperature of the heated elements, K
$\Delta T'$	temperature difference, $\Delta T' = T'_H - T'_C$
T_r	dimensionless reference temperature, $T_r = T'_C/(T'_H - T'_C)$
(u, v)	dimensionless horizontal and vertical velocities, $(u, v) = (u', v')H'/\alpha$
(x, y)	dimensionless coordinates, $(x, y) = (x', y')/H'$

Greek symbols

α	thermal diffusivity of fluid, m^2/s
β	thermal expansion coefficient of fluid, 1/K
ε_1	emissivity of the active elements
ε_2	emissivity of the insulated elements
λ	thermal conductivity of fluid, W /(K $\times$ m)
ν	kinematic viscosity of fluid, m^2/s

Ω dimensionless vorticity, $\Omega = \Omega' H'^2 / \alpha$
Ψ dimensionless stream function, $\Psi = \Psi' / \alpha$
σ Stéfan-Boltzman constant, $\sigma = 5.6697\ 10^{-8}$ W / ($m^2 \times K^4$)

Subscripts
B bottom
T top
V vertical

Superscripts
$'$ dimensional variable

1. INTRODUCTION

In the literature, most of the existing studies on the coupling between natural convection and thermal radiation are concerned with rectangular cavities where the temperature gradient is either horizontal or vertical, including different kinds of boundary conditions [1-6]. Results of these studies show that radiation affects the dynamical and thermal structures of the fluid, reduces the natural-convective heat transfer component, and contributes to increasing the total amount of heat exchanged in the configurations considered. Most of the works conducted in the past on natural convection coupled with radiation inside rectangular enclosures has been substantially oriented to study unidirectional heat flows resulting from imposing heat flux or temperature gradients either parallel or normal to gravity.
In some practical situations (convective heat losses from solar collectors, thermal design of buildings, air conditioning and recently, the cooling of electronic components), much more complex boundary conditions may be encountered where horizontal and vertical temperature gradients can be simultaneously imposed across the cavity. This justifies the presence of published works where the rectangular cavities are heated from below and cooled from above and simultaneously submitted to various specified thermal boundary conditions at the sidewalls. In the absence of radiation effect, Corcione [7] has investigated numerically steady 2D laminar natural convection of air-filled rectangular enclosures which are heated from below, cooled from above and submitted to various thermal boundary conditions at the sidewalls. Heat transfer correlation equations have been derived for each of the thermal boundary conditions imposed to the studied configurations.With respect to the basic thermal configuration wherein the sidewalls are insulated, the heat transfer rate resulting from the heated bottom wall increases when each adiabatic sidewall is replaced by a cooled sidewall, while showing only slight decreases as such replacement is carried out through a heated sidewall.
Cianfrini and al. [8] have studied numerically natural convection heat and momentum transfer in air-filled square enclosures with differentially heated opposite walls and inclined with respect to the gravity vector. The study is conducted for various Rayleigh numbers and inclination angles of the cavity. The results obtained for the average Nusselt number of the whole cavity were expressed through a semi-empirical dimensionless correlation.
According to the literature review, there exist scarce research works on natural convection coupled with radiation in a square cavity submitted to cross gradients of temperature. Therefore, to help address the lack of works on the cross gradients of thermal boundary conditions, the aim of the present study is to analyze numerically the coupling of natural convection and thermal radiation in a square cavity submitted to cross gradients of temperature. Combined effects of the Rayleigh number, Ra, the emissivity of the walls, ε, and the relative length of the active walls, B, on the dynamical and thermal behaviors are analyzed. The contribution of convection and radiation to the overall heat transfer is also quantified.

2. PROBLEM FORMULATION

The configurations under study, together with the system of coordinates, are depicted in Figs. 1a-1b.

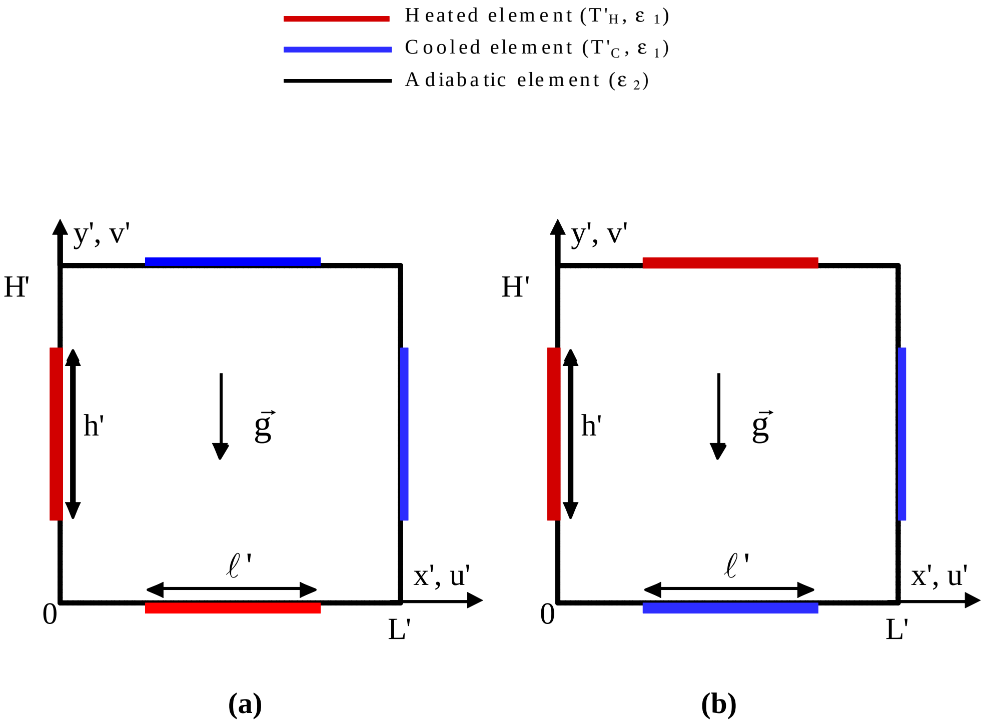

Figure 1: Geometry of the problem with the imposed thermal excitations.

It consists of a square cavity which is discretely heated and cooled from the four walls, with the assumption that the horizontal and vertical temperature differences are the same. The *VB* (Fig. 1a) / *VT* (Fig. 1b) configurations corresponding to vertical-bottom / vertical-top heated walls are examined. They correspond to a destabilizing (*VB*) and stabilizing (*VT*) situations, respectively. The inner surfaces, in contact with the fluid, are assumed to be gray, diffuse emitters and reflectors of radiation. The third dimension of the cavity (direction perpendicular to the plane of the diagram) is assumed to be large enough so that the fluid flow can be considered two-dimensional. The flow is conceived to be laminar and incompressible with negligible viscous dissipation. All the thermophysical properties of the fluid are assumed constant except the density in the buoyancy term which is assumed to vary linearly with temperature (Boussinesq approximation); such a variation gives rise to the buoyancy forces. Taking into account the above-mentioned assumptions, the non-dimensional governing equations, written in vorticity-stream function (Ω-Ψ) formulation, are as follows:

$$\frac{\partial \Omega}{\partial t}+\frac{\partial (u\Omega)}{\partial x}+\frac{\partial (v\Omega)}{\partial y}=\mathrm{R}a\,\mathrm{Pr}\frac{\partial T}{\partial x}+\mathrm{Pr}\left(\frac{\partial^2 \Omega}{\partial x^2}+\frac{\partial^2 \Omega}{\partial y^2}\right) \tag{1}$$

$$\frac{\partial T}{\partial t}+\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}=-\Omega \quad (3)$$

The dimensionless stream function and the vorticity are related to the non-dimensional velocity components by the following expressions:

$$u=\frac{\partial\Psi}{\partial y}\ ,\ v=-\frac{\partial\Psi}{\partial x}\ \text{and}\qquad \Omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \quad (4)$$

2.1 Boundary conditions

The dimensionless boundary conditions associated to the problem are:

$u = v = \Psi = 0$	on the cavity walls
$T = 0$	on the cooled elements
$T = 1$	on the heated elements
$-\frac{\partial T}{\partial n}+N_r Q_r=0$	on the adiabatic elements

″n″ being the normal direction to the considered adiabatic wall.

2.2 Surface radiation calculations

The calculation of the radiative heat exchange between the internal walls of the cavity is based on the radiosity method. The grid used for convection (81×81) is retained in the presence of radiation and consists of 324 isothermal elementary segments. Each segment is sufficiently short to be considered isothermal. The view factors between the isothermal elementary surfaces were determined by Hottel's [9] crossed string method. The summation rules were checked to be sure that the view factors F_{ij} summation equals unity for each surface. The inner surfaces of the enclosure are assumed to be opaque, diffuse and gray. The non-dimensional radiosity equation for the i^{th} element of the enclosure, in the case of a radiatively non-participating medium, may be written as:

$$J_i=\varepsilon_i\left(\frac{T_i}{T_r}+1\right)^4+(1-\varepsilon_i)\sum_{S_j}F_{ij}\,J_j \quad (5)$$

The non-dimensional net radiative heat flux leaving a surface S_i is evaluated by:

$$Q_r=\varepsilon_i\left[\left(\frac{T_i}{T_r}+1\right)^4-\sum_{S_j}F_{ij}\,J_j\right] \quad (6)$$

2.3 Heat transfer

The average Nusselt numbers, characterizing the contributions of natural convection and thermal radiation to the overall heat transfer through the heated walls, are respectively defined as:

- On the left vertical heated wall:

$$Nu_V(cv) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial x}\right)\Bigg|_{x=0} dy \quad ; \quad Nu_V(rd) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r\right)\Big|_{x=0} dy \tag{7}$$

On the lower horizontal heated wall:

$$Nu_B(cv) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial y}\right)\Bigg|_{y=0} dx \quad ; \quad Nu_B(rd) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r\right)\Big|_{y=0} dx \tag{8}$$

- On the horizontal top heated wall:

$$Nu_T(cv) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial y}\right)\Bigg|_{y=1} dx \quad ; \quad Nu_T(rd) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r\right)\Big|_{y=1} dx \tag{9}$$

Average convective and radiative Nusselt numbers across the whole cavity are defined respectively as:

- For *VB* configuration

$$Nu(cv) = Nu_V(cv) + Nu_B(cv)$$

and

$$Nu(rd) = Nu_V(rd) + Nu_B(rd)$$

- For *VT* configuration

$$Nu(cv) = Nu_V(cv) + Nu_T(cv)$$

and

$$Nu(rd) = Nu_V(rd) + Nu_T(rd)$$

The mean Nusselt number, Nu, characterizing the global heat exchange between the walls of the cavity and the confined fluid is calculated as:

$$Nu = Nu(cv) + Nu(rd).$$

Where Nu(cv) and Nu(rd) correspond to the convective and radiative Nusselt numbers obtained for each of the two configurations.

3. METHOD OF SOLUTION

The non linear partial differential governing equations (1)-(3) are discretized with a second order accurate finite difference method. The integration of equations (1) and (2) is ensured by the Alternate Direction Implicit method (ADI). Values of the stream function at all grid points are obtained with Eq. (3) by using the Point Successive Over-Relaxation method (PSOR) with an optimum over-relaxation coefficient equal to 1.92 for the grid 81×81 adopted in the present study. A convergence criterion is adopted for the stream function to satisfy a variation by less than 10^{-5} at each time step. A further decrease of this value does not cause any significant change in the results. The velocities at all grid points are determined with equation (4) using up-dated values of the stream function. The set of equations (5), representing the radiative heat transfer between the different elementary surfaces of the cavity, is solved by using the Gauss-Seidel method. The numerical code is validated against the results of Akiyama and Chong [2] obtained in the case of a square cavity differentially heated. Results of comparisons, made in terms of convective Nusselt numbers, evaluated at the heated wall and presented in Table 1, show a fairly good agreement with relative maximum deviations limited to 1.07 % / (1.36 %) for $\varepsilon = 0$ / (1) for Ra varying in the range $10^3 \le Ra \le 10^6$. The accuracy of the numerical model is checked by comparing results from the present investigation against those previously published by De Vahl Davis [10] in the case of a differentially heated square cavity. Comparative results, presented in Table 2, show an excellent agreement with maximum deviations of about 0.85% and 2.9% respectively in terms of Ψ_{max} and Nu. Finally, we proceed to a supplementary test of validation by checking the energy balance of the system. In fact, it is carefully verified that the energy transmitted to the fluid by the heated walls leaves the cavity through the cold ones; this energy balance is verified for all the computations conducted with a maximum difference of 1 %.

	$\varepsilon = 0$				$\varepsilon = 1$			
Ra	10^3	10^4	10^5	10^6	10^3	10^4	10^5	10^6
Present work	1.118	2.257	4.627	9.475	1.250	2.242	4.192	8.100
Akiyama and Chong [2]	1.125	2.250	4.625	9.375	1.250	2.250	4.250	8.125

Table 1: Effect of Ra and ε on the mean convective Nusselt number, Nu(cv), evaluated on the heating wall of a square cavity for T'_H = 298.5 K and T'_C = 288.5 K.

	Present study		De Vahl Davis [10]	
Ra	Ψ_{max}	*Nu*	Ψ_{max}	*Nu*
10^3	1.173	1.118	1.174	1.116
10^4	5.070	2.257	5.098	2.242
10^5	9.656	4.639	9.739	4.564
10^6	17.464	9.520	17.613	9.270

Table 2: Validation of the numerical code with the results of De Vahl Davis [10].

4. RESULTS AND DISCUSSION

In convective heat transfer problems where the thermal radiation is considered, the Rayleigh number, Ra, the convection-radiation interaction number, N_r, and the dimensionless reference temperature, T_r, appear among the main controlling parameters but these three parameters cannot be varied simultaneously. In the present study, the temperature of the cold elements is maintained constant at $T'_C = 298.15\,K$. The variation of the Rayleigh number induces automatic variations of the parameters N_r and T_r. The emissivities of the active and adiabatic walls are respectively ε_1 and ε_2. In the following, and for the *VB* and *VT* heating modes, effects of Rayleigh number, $10^4 \le Ra \le 10^7$, and walls emissivity $0 \le \varepsilon \le 1$, ($\varepsilon_1 = \varepsilon_2 = \varepsilon$), on fluid flow and heat transfer characteristics are illustrated.

In practice, it is very difficult to get surface's coatings having ε = 0 and 1 (theoretical limits of the emissivity). The use of a highly polished aluminum sheet and a black paint correspond to emissivities of 0.05 and 0.85, respectively. The calculations are performed by considering air as a working fluid (Pr = 0.72). The relative length of the active elements is fixed at B = 0.5.

4.1 Effect of Ra on the stream functions Ψ_{min} and Ψ_{max}

The variations of the minimum stream function Ψ_{min}, characterizing the intensity of the main flow, with Rayleigh number, Ra, is illustrated in Fig. 2a for different values of the emissivity of the walls. This figure includes the results corresponding to the two heating modes *VB* and *VT*.

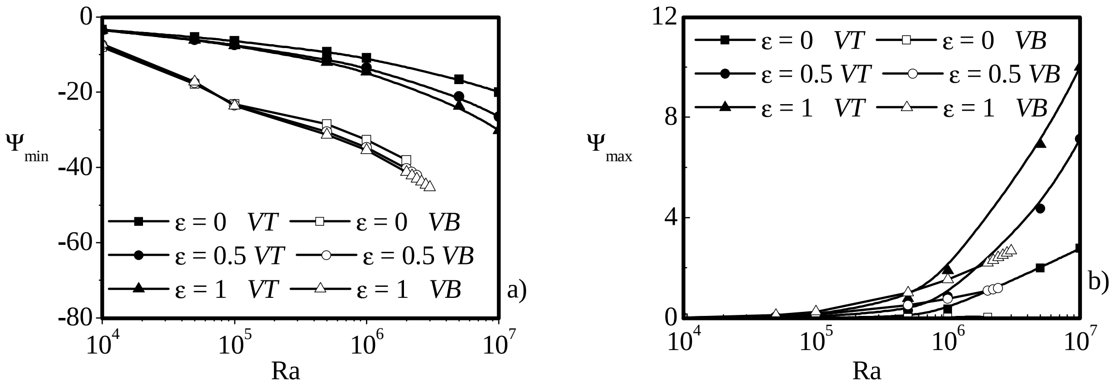

Figure 2: Effect of Ra, for *VB* and *VT* modes, on the fluid flow intensity for B = 0.5 and various values of ε: (a) Ψ_{min} and (b) Ψ_{max}.

For all values of ε, the increase of Ra leads to a monotonous increase of $|\Psi_{min}|$, indicating an important intensification of the flow with Ra. For a given Ra and in the case of *VT* heating mode, the increase of the emissivity engenders an acceleration of the fluid circulation (increase of $|\Psi_{min}|$), more pronounced for the large values of Ra. Fig. 2a shows an insensitivity of Ψ_{min} with respect to the variation of ε. These behaviors are, for both *VB* and *VT*, different from that observed in the case of a cavity differentially heated or submitted to a vertical gradient of temperature (the increase of the walls' emissivity slows down the fluid). Variations of the maximum stream function Ψ_{max}, characterizing the intensity of the secondary flow, are illustrated in Fig. 2b. For the two heating modes Ψ_{max} remains insensitive to the increase of Ra and ε whatever the value of Ra ≤ 10^5. Beyond this value, an increase of Ψ_{max} with Ra, more pronounced when ε increases, is observed. Note that, in general, the importance of the secondary flow is supported by the emissivity of the walls but it is insignificant in the absence of radiation or in the case of polished walls.

4.2 Effect of Ra on Nusselt numbers

For the two heating modes *VB* and *VT*, variations, versus the Rayleigh number Ra, of the average Nusselt numbers, resulting from contributions of convection and radiation and the total Nusselt number, evaluated along the heated elements, are presented in Figs. 3a–d for Ra = 10^6 and various values of ε.

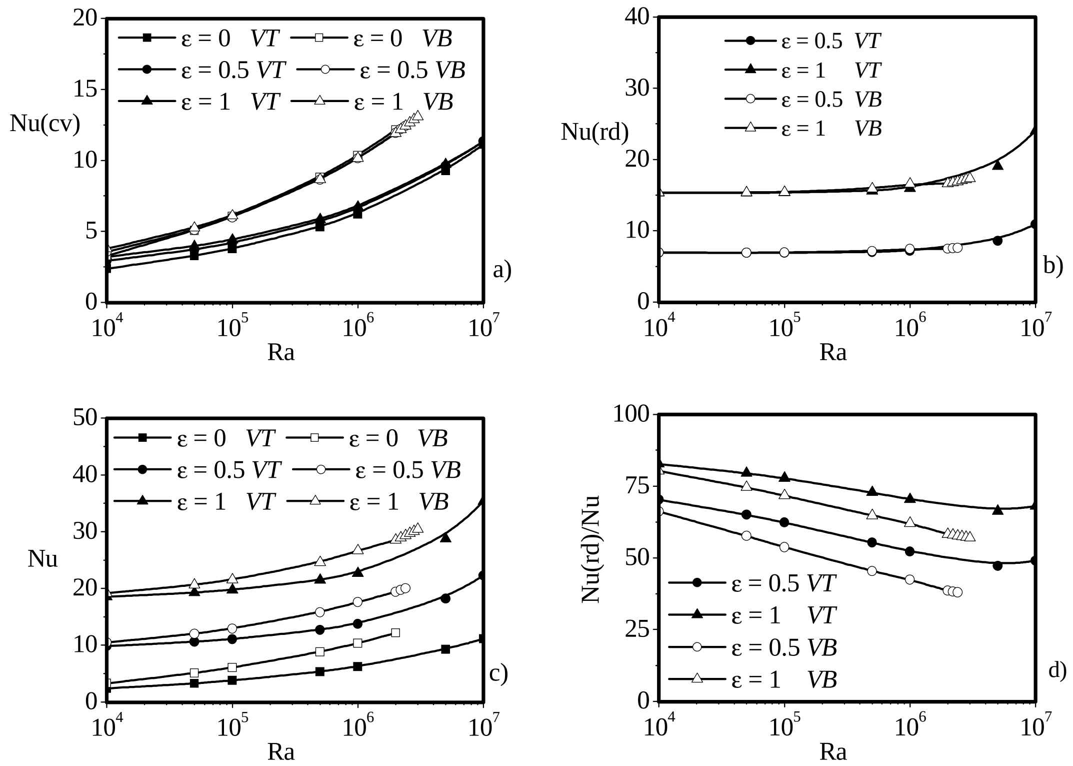

Figure 3 : Variations, with Ra, for *VB* and *VT* modes of Nusselt numbers for B = 0.5 and various values of ε : a) Nu(cv), b) Nu(rd), c) Nu and d) Nu(rd)/Nu.

Figure. 3a shows a monotonous increase of Nu(cv) with Ra either for *VB* or *VT* modes. This tendency is due to the flow intensification promoted by the increase of Ra. It can be seen that the rate of convection heat transfer is larger for the *VB* configuration than that of the *VT* one. This result can be explained by the fact that in the case of *VB* heating mode the upward imposed vertical gradient of temperature has an destabilizing effect on the velocity and temperature fields, and contributes to an important intensification of the flow circulation (Ψ_{min} = –35.36 for ε = 1) and consequently a high convective Nu. But, in the *VT* mode, the stabilizing downward imposed vertical temperature gradient drives to a slowdown of the fluid flow intensity inside the cavity (Ψ_{min} = –14.58 for ε = 1) leading to convective heat transfer lower than the *VB* mode. Also, in the case of *VB* configuration and for all considered values of Ra, the convective Nusselt number remains practically insensitive to the increase of the walls' emissivity and therefore not affected by the effect of radiation. This finding is fundamentally different from tendencies characterized by the well known negative role of radiation on the natural convection heat transfer component [1, 2, 6]. A close examination of the variations of the convective Nusselt numbers on the vertical and horizontal heated walls shows that the effect of surface radiation is different on these walls. In fact, results (not presented here) show that surface radiation leads to an enhancement/decrease in the convective component across the vertical/horizontal heated walls. These contrary tendencies tend to be compensated since the dual effects of radiation on the two heated walls cancel each others. Note that, previous results reported in the literature on the interaction between natural convection and surface radiation in rectangular enclosures submitted to a single temperature gradient (horizontal or vertical), show that surface radiation leads to a decrease of the convective Nusselt number [1, 2, 6]. It is important to note that for the *VT* heating mode, Fig. 3a shows an increase of Nu(cv) with ε. This is due to the fact that in this mode, the heat radiation tends to increase the walls temperature of the cavity and reduce the negative effect of the downward imposed vertical gradient of temperature.

For the two heating modes *VB* and *VT*, the effect of the emissivity of the walls on the radiative heat transfer component is presented in Figure 3b in terms of Nu(rd) variations with Ra for various ε. Globally, it can be deduced that, for a given value of Ra, the rate of radiative heat transfer increases importantly by increasing ε. However, the variation of the convective Nusselt number, the radiative heat transfer is globally less affected by the increase of Rayleigh number for all values of the walls emissivity. The variations of the total Nusselt number with Ra, presented in Figure 3c, show increasing tendencies of Nu with Ra and ε; this behavior is explained by the increase of its convective component with Ra and its radiative component with ε.

To quantify the contribution of radiation to the total heat transfer through the cavity, we present in Figure 3d the evolution of the ratio Nu(rd)/Nu with Ra for various values of ε. It can be seen that the contribution of radiation decreases almost linearly with Ra for all values of ε. Hence, for each value of ε, the contribution of radiation is maximum for the lowest value of Ra and minimum for the highest value of this parameter. All these drops in the contribution of the radiative component to the total Nusselt number are attributed to the increasing magnitude of the convection fluid flow with the buoyancy effect, promoted by the increase of Ra. This leads to more vigorous heat transfer by convection, and consequently, to a reduction of the contribution of radiation to the overall heat transfer. It is worth pointing out that for the *VT* configuration, this contribution is higher than that of the *VB* one. In fact, for the later, the rate of heat transfer by convection is important, leading to a reduction of the contribution of radiation to the total heat transfer. Quantitatively, for ε = 1 and for mode *VB* / (*VT)*, radiation of the walls contributes by about 80,279%/(82.68%) for Ra = 10^4 and this contribution drops to about 62.01%/(70,36%) for Ra = 10^6. The plots of Nu vs. Ra show clearly that these drops in percentage of the radiative component contribution, engendered by the increase of Ra, are largely compensated by the increase of the convective component since the latter is largely promoted by the buoyancy force. For $\varepsilon \geq 0.5$, the contribution of radiation component remains larger than 50% for *VT* heating mode. However, in the case of *VB* mode, for ε = 0.5, the contribution of radiation is prevalent for values of $Ra \leq 2\times10^5$.

4.3 Effect of the emissivities ε_1 and ε_2

In order to investigate the effect of different emissivities of the walls, four relevant cases are considered based on the respective emissivities ε_1 and ε_2 of the active and insulated walls. These different cases are summarised in Table 3. In practice, the coatings lead to surfaces with emissivities ε different from 0 and 1 (theoretical limits of the emissivity). The use of a highly polished aluminum sheet and a black paint correspond to emissivities of 0.05 and 0.85, respectively.

Case	ε_1	ε_2
1	0.05	0.85
2	0.85	0.05
3	0.05	0.05
4	0.85	0.85

Table 3: Wall emissivities used for each case

For the two heating modes *VB* and *VT*, Fig. 4/Figure 5 display the variations, with Ra, of the average Nusselt numbers evaluated at the heated walls in the cases 1 and 3 (ε_1 = 0.05)/cases 2 and 4 (ε_1 = 0.85).
It can be seen from Figure 4a and Figure 5a that the convective heat transfer increases by increasing Ra for the two heating modes and for all considered cases without significant effect of the emissivity of the walls. However, Nu(cv) corresponding to *VB* mode is larger than *VT* one, results due to the magnitude of flow intensity which is important for *VB* mode. The evolution of the radiative average Nusselt number with respect to changes in walls emissivities and Ra is presented in Figure 4b and Figure 5b respectively for cases 1 and 3 and cases 2 and 4. We can note that, for the *VB* and *VT* heating modes, the radiative heat flux is important when the emissivity of the active walls is higher (ε_1 = 0.85), this corresponds to cases 2 and 4 (Figure 5b). For the cases 1 and 3 (ε_1 = 0.05), Figure 4b shows that the contribution of radiation remains almost constant at very small values. In fact, the contribution of radiative heat transfer is highly affected by the active walls emissivity. For a given value of ε_1 and Ra, the transfer rate by radiation remains constant whatever the value of the emissivity, ε_2, of the insulated walls is. The evolution of the total Nusselt number Nu is presented in Figure 4c and Figure 5c. It is to be noted that for cases 1 and 3 (Figure 4c), Nu remains close Nu(cv) for all values of Rayleigh number whatever the value of ε_2 and confirms that the effect of ε_1 contributes to the enhancement of the heat transfer across the enclosure. The variation with Ra of the contribution of radiation to the overall heat transfer across the enclosure is quantified for each case and presented in Figure 4d and Figure 5d. For the *VB* heating mode, It is seen that in cases 1 and 3 (Figure 4d) the radiation contribution remains generally weak (5.4 % ≤ Nu(rd)/Nu ≤ 16 %), in contrast, the contribution of radiation for cases 2 and 4 (Figure 5d) is important (52 ≤ Nu(rd)/Nu ≤ 78.8 %).

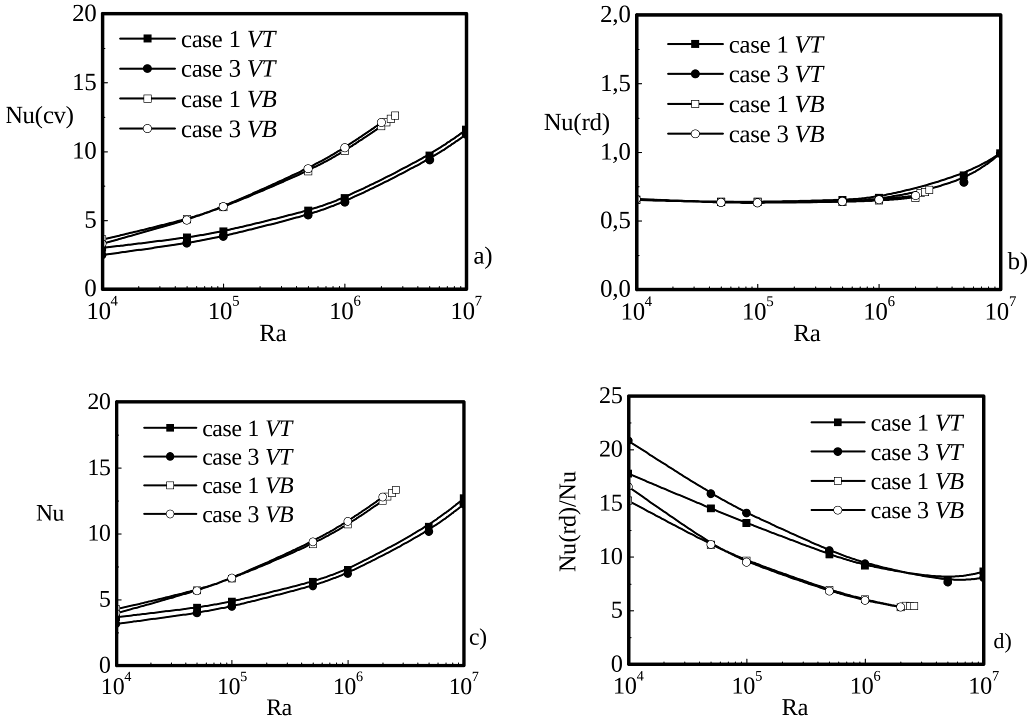

Figure 4 : Variations, for *VB* and *VT* modes, with Ra, of Nusselt numbers for cases 1 and 3 (B = 0.5): a) Nu(cv), b) Nu(rd), c) Nu and d) Nu(rd)/Nu.

In the other mode, *VT*, for cases 1 and 3 Fig. 4d gives 8.1 % ≤ Nu(rd)/Nu ≤ 20.8 %, and Fig. 5d gives 60 % ≤ Nu(rd)/Nu ≤ 83.13 % for cases 2 and 4. These results show that the heat transfer is dominated by convection for cases 1 and 3 while it is dominated by radiation for cases 2 and 4. Finally, we can note that the contribution of radiation in the *VT* heating mode is larger than that in the *VB* one, due that Nu(cv) is more important in *VB* than *VT*.

5. CONCLUSION

The problem of two-dimensional combined natural convection and surface radiation inside a square cavity, submitted to cross gradients of temperature, are studied numerically. Two heating mode, *VB* and *VT*, are considered. In *VB*, the bottom active element is at a higher temperature than that of the top active one. The *VT* corresponds to the reverse situation. Combined effects of the emissivity ε and the Rayleigh number, Ra, on heat transfer and flow structure are examined. Both cases of pure natural convection and natural convection coupled with radiation are considered. It can conclude that in the case of *VB* heating mode the upward imposed vertical gradient of temperature has a destabilizing effect on the velocity and temperature fields, and contributes to an important intensification of the flow circulation and consequently a high convective Nu. However, in the *VT* mode, the stabilizing downward imposed vertical temperature gradient drives to a slowdown of the fluid flow intensity inside the cavity leading to less convective heat transfer. It is found that the radiation has no effect on the average convective Nusselt number component and the latter is supported by the increase of the

Rayleigh number Ra. When $\varepsilon_1 = \varepsilon_2 = \varepsilon$, it is found that the parameter ε has a positive effect on the radiative and total heat transfers. The contribution of radiation to the total heat transfer is generally not negligible for $\varepsilon \geq 0.5$.

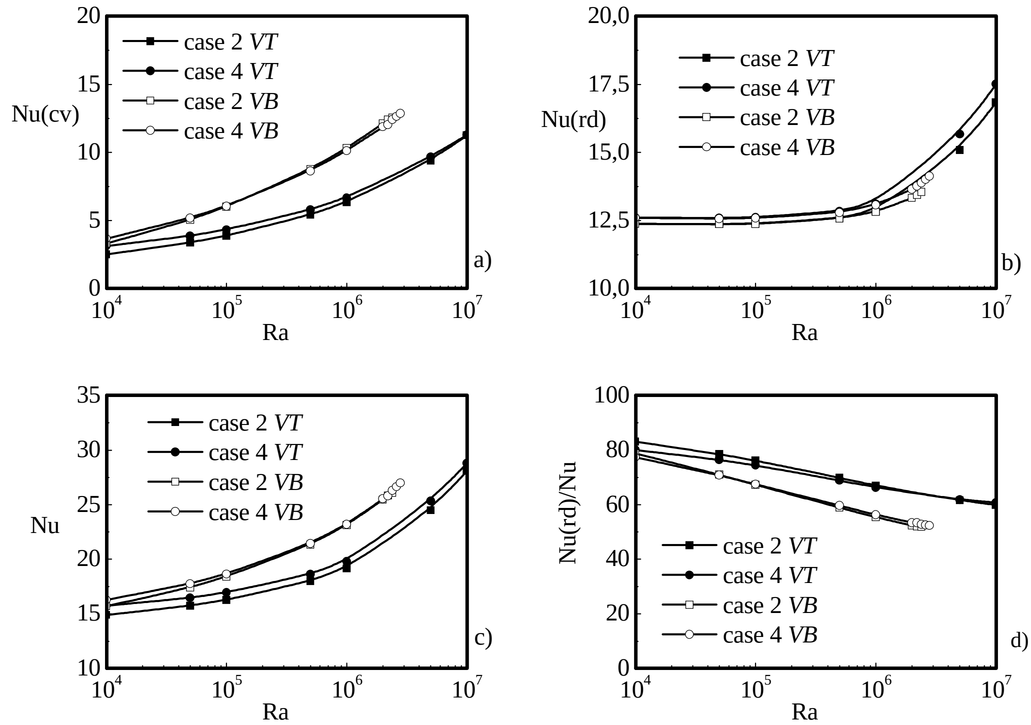

Figure 5 : Variations, for *VB* and *VT* modes, with Ra, of Nusselt numbers for cases 2 and 4 (B = 0.5): a) Nu(cv), b) Nu(rd), c) Nu and d) Nu(rd)/Nu.

6. REFERENCES

1. C. Balaji and S. P. Venkateshan, Interaction of surface radiation with free convection in a square cavity, Int. J. Heat Fluid Flow **14** (1993), pp. 260–267.
2. M. Akiyama and Q.P. Chong, Numerical analysis of natural convection with surface radiation in a square cavity, Num. Heat Transfer Part A **31** (1997), pp. 419–433.
3. G. Colomer, M. Costa, R. Consul and A. Oliva, Three dimensional numerical simulation of convection and radiation in a differentially heated cavity using the discrete ordinates method. Int. J. Heat Mass Transfer **47** (2004), pp. 257–269.
4. C. Y. Han and S. W. Baek, The Effects of Radiation on Natural Convection in a Rectangular Enclosure Divided by Two Partitions, Num. Heat Transfer Part A **37** (2000), pp. 249–270.
5. E.H. Ridouane, M. Hasnaoui, A. Amahmid and A. Raji, Interaction between natural convection and radiation in a square cavity heated from below, Num. Heat Transfer Part A **45** (2004), pp. 289–311.
6. R. El Ayachi, A. Raji, M. Hasnaoui and A. Bahlaoui, Combined effect of radiation and natural convection in a square cavity differentially heated with a periodic temperature, Num. Heat Transfer Part A **53** (2008), pp. 1339–1356.

7. M. Corcione, Effects of the thermal boundary conditions at the sidewalls upon natural convection in rectangular enclosures heated from below and cooled from above, Int. J. of Thermal Sciences **42** (2003), pp. 199–208.
8. C. Cianfrini, M. Corcione and P. P. Dell'Omo, Natural convection in tilted square cavities with differentially heated opposite walls, Int. J. Thermal Sciences **44** (2005), pp. 441–451.
9. H. C. Hottel and A. F. Saroffim, Radiative heat transfer, McGraw-Hill, New York, 1967.
10. G. De Vahl Davis, Natural convection of air in a square cavity: a Bench-Mark numerical solution, Int. J. Num. Meth. Fluids **3** (1983), pp. 249–264.