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An Inverse Problem Approach for the Identification of Illegal Groundwater Pumping in 2D Aquifers

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Abstract
A methodology for identifying illegal groundwater pumping wells in semi-confined aquifers is presented. The procedure is formulated within an optimisation framework by properly adjusting the unknown illegal well magnitudes in order to make the simulated head achieves the best reproduction possible of that sampled. The optimal pumping schedule is approached iteratively by applying the Levenberg-Marquardt algorithm and influence coefficient simulator, in order to minimize the Euclidean distance between simulated and observed heads. The methodology has been successfully applied to a constructed example of 2-D aquifer.

Résumé
Une méthodologie pour l'identification des pompages illégaux des eaux souterraines dans les aquifères semi-confinés est exposée. Elle repose sur une procédure d'optimisation en reproduisant le plus fidèlement possible le potentiel hydraulique échantillonné à partir d'un certain nombre de puits d'observation, et ce en ajustant l'intensité de pompage des puits illégaux. Le scénario optimal de pompage est approché de manière itérative en combinant l'algorithme de Levenberg-Marquardt et un simulateur à base des coefficients d'influences, et ce dans le but de minimiser la distance Euclidienne entre les potentiels hydrauliques simulé et observé. La méthode a été appliquée avec succès à un exemple synthétique de nappe bidimensionnelle.

1. INTRODUCTION

The identification of groundwater illegal pumping remains among essential measures for a fair groundwater sharing. Suspicion may develop for instance around agricultural or industrial areas, such as private farms and factories where water abstraction or wastewater injection is probable. Inquiry of such "water crime" is typically the job of the "Water Police" (Police de l'Eau) as recently introduced by Moroccan legislation.

This work suggests a methodology combining calibrated numerical models and observations. It extends the problem formulated and solved for a one-dimensional aquifer model by Saffi & Cheddadi [1]. From the governing equation viewpoint the recovery of the source term lies within inverse problem category. So, given a calibrated direct code and a record of observed hydraulic heads, the point is to arrive at such well configuration (locations and magnitudes) which gives the best possible reproduction of the monitored hydraulic head.

2. PROBLEM STATEMENT

The mathematical model considered in this paper rules 2D semi-confined flow. It writes in a flow region $\boldsymbol{\Omega}$ [2]:

$$\nabla.(T\nabla h)+\frac{K_a}{m_a}(H_a-h)=\bar{S}\,\frac{\partial h}{\partial t} \tag{1}$$

Where h is the hydraulic head, T is the aquifer transmissivity, K_a and m_a are respectively the conductivity and the thickness of the layer overlying the aquifer. H_a is the external hydraulic head. $\bar{S}$ is the storage coefficient.

In this work two kinds of wells are distinguished. Legal wells (*LW*) are known in number, location and magnitude. In the following $\mathbf{Q}(t)$ denotes the $\bar{q}$-vector $(Q_{LW_1}(t), Q_{LW_2}(t),..., Q_{LW_{\bar{q}}}(t))^T$ with $Q_{LW_j}(t)$ being the magnitude of the *j*th legal well and $\bar{q}$ is the total number of these wells.

Illegal wells refer to the areas from which water may be abstracted from or injected into the aquifer without prior permission. So, they are neither known in number nor in magnitude or location. That is, it will be spoken of potential illegal well (*PIW*) as to indicate the $\bar{r}$ locations that are thought to be subjected to illegal pumping. If $P_{PIW_j}(t)$ denotes the hypothetical magnitude of the *j*th potential illegal well then a parameter vector $\mathbf{P}(t)$ is introduced as the transpose of $(P_{PIW_1}(t), P_{PIW_2}(t),..., P_{PIW_{\bar{r}}}(t))$. $\mathbf{P}(t)$ is the problem unknown that needs to be determined.

Given a well field configuration $(\mathbf{P}(t),\mathbf{Q}(t))$, the hydraulic head distribution $\mathbf{h}(t)$ over the flow region $\boldsymbol{\Omega}$ is solution to the following dynamic equation [3]:

$$\begin{cases} \mathbf{A}\dfrac{d\mathbf{h}}{dt}+\mathbf{Bh}+\mathbf{F}(\mathbf{P},\mathbf{Q}) \;=\; \mathbf{0} \\ \qquad\qquad \mathbf{h}(0) \;=\; \mathbf{h}^{(0)} \end{cases} \tag{2}$$

where $\mathbf{h}(t)=(h_1(t),h_2(t),...,h_n(t))^T$. $h_j(t)$ is the hydraulic head in node number *j* and at time *t*. *n* is the number of discretization nodes. $\mathbf{h}^{(0)}$ is the vector of initial heads. $\mathbf{A}$ and $\mathbf{B}$ are ($n\times n$)-matrices written in terms of the aquifer storativity and transmissivity, respectively. $\mathbf{F}$ is the forcing vector containing point and distributed sources/sinks and the boundary conditions. $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{F}$ are computed for 2D semi-confined flow equation discretized using linear triangular finite elements.

Equation (2) can also be presented as [4]:

$$\mathbf{h}^{(s)}=\bar{\mathbf{h}}^{(s)}(\mathbf{Q})+\mathbf{G}^{(s)}\mathbf{P}.\Delta t \qquad \text{for} \quad s\geq 1 \tag{3}$$

Where $\mathbf{h}^{(s)} = (h_1^{(s)}, h_2^{(s)}, ..., h_n^{(s)})^T$. $h_j^{(s)}$ denotes the hydraulic head in node *j* and at time level *s*. $\overline{\mathbf{h}}^{(s)}$ is the vector of nodal hydraulic head when all illegal wells are inactive, $\mathbf{Q} = (Q_{LW_1}^{(1)}, Q_{LW_1}^{(2)}, ..., Q_{LW_1}^{(s_{\max})}; Q_{LW_2}^{(1)}, Q_{LW_2}^{(2)}, ..., Q_{LW_2}^{(s_{\max})}; ...; Q_{LW_{\bar{q}}}^{(1)}, Q_{LW_{\bar{q}}}^{(2)}, ..., Q_{LW_{\bar{q}}}^{(s_{\max})})^T$ with $Q_{LW_j}^{(k)}$ being the pumping rate in the *LW* number *j* at the time level *k* and $\mathbf{P} = (P_{PIW_1}^{(1)}, P_{PIW_1}^{(2)}, ..., P_{PIW_1}^{(s_{\max})}; P_{PIW_2}^{(1)}, P_{PIW_2}^{(2)}, ..., P_{PIW_2}^{(s_{\max})}; ...; P_{PIW_{\bar{r}}}^{(1)}, P_{PIW_{\bar{r}}}^{(2)}, ..., P_{PIW_{\bar{r}}}^{(s_{\max})})^T$ with $P_{PIW_j}^{(k)}$ being the unknown magnitude of the *PIW* number *j* during the time level *k*. $s_{\max}$ is the number of time levels over which the system is considered. Δt is the discretization time step.

$\mathbf{G}^{(s)}$ stands for the matrix of instantaneous influence coefficients at time *s*, computed for 2D semi-confined flow using triangular finite element [4]. Detailed procedure to compute influence coefficients is found in [1,5]. The general term $G_{p,PIW_k}^{s,i}$, of matrix $\mathbf{G}^{(s)}$, is read as the aquifer response at time level *s*, in node *p* to a unit stress applied at time level *i* in the PIW_k under a set of homogeneous boundary and initial conditions.

The following development seeks to adjust the model input $\mathbf{P}$ in order to achieve a satisfactory degree of correspondence between $\mathbf{h}^{(s)}$ given by equation (3) and its counterpart $\mathbf{h}^{obs}$, observed over a set of $\overline{m}$ monitoring wells MW_i.

3. IDENTIFICATION PROCEDURE

The aquifer system is observed through $\overline{m}$ monitoring wells MW_1, MW_2,…, $MW_{\overline{m}}$ over a period of time $[s_{\min}, s_{\max}]$. $s_{\min}$ corresponds to the time step when the first head observation is made. $[h_{MW_j}^{\text{obs}}]^{(s)}$ denotes the observed head at the monitoring well MW_j and the time step s $1 \le j \le \overline{m}$ and $s_{\min} \le s \le s_{\max}$. The simulated hydraulic head at MW_j and time step *s*, i.e. the compoment $h_{MW_j}^{(s)}$ of the vector $\boldsymbol{h}^{(s)}$ of equation (3) will be contrastly denoted $[h_{MW_j}^{\text{sim}}]^{(s)}$.

The adjustment of $\mathbf{P}$ can be carried out either using equation error criterion or output error criterion methods for groundwater inverse problem solving as classified by Yeh [6].

The identification procedure used here consists in minimizing the conventional least square criterion J which measures the misfit between observed and simulated heads:

$$\operatorname{Min}_{\mathbf{P}} \ J(\mathbf{P}) = \sum_{s=s_{\min}}^{s_{\max}} \sum_{i=1}^{\overline{m}} \left(\left[h_{MW_i}^{sim} \right]^{(s)} (\mathbf{P}) - \left[h_{MW_i}^{obs} \right]^{(s)} \right)^2 \tag{4}$$

While solving (4), the optimisation algorithm calls repeatedly the the forward code to compute $\left[h_{MW_i}^{sim}\right]^{(s)}$ for feasible $\mathbf{P}$ vectors until a stopping criteria is met. So the CPU time lasted to compute $\left[h_{MW_i}^{sim}\right]^{(s)}$ is crucial to output error criterion method. In fact, $\left[h_{MW_i}^{sim}\right]^{(s)}$ can be extracted directly by solving equation (2). In this case a flow equation is to be played anew whenever the optimization engine request is met. This defines the basic function behind conventional simulators [7]. And, the major limitation of this approach is that a great computational burden is unnecessarily redone during criterion *J* minimization. The second alternative to obtain $\left[h_{MW_i}^{sim}\right]^{(s)}$, is to use an influence coefficient simulator. This latter is by far more time saving since it does not solve the flow equation but convolutes the influence coefficient records $\mathbf{G}^{(s)}$ with the excitation set $\mathbf{P}$ in accordance with equation (2). So, influence coefficients that were implicitly generated for every $[\mathbf{h}^{sim}]^{(s)}$ call, are now computed for once and all ahead of the minimization work [3].

4. CONSTRUCTED EXAMPLE : 2-D AQUIFER

The methodology exposed in this paper is applied to a constructed example of 2-D aquifer inspired from [2]. Figure 1a displays a hypothetical flow region Ω delimited by two river reaches that impose a constant head of 100 m. The two other (shaded) sections of Ω border are of no-flow type. The aquifer transmissivity and storage coefficient are $1000\,\mathrm{m^2d^{-1}}$ and 0.01 respectively. Semi-confining layer hydraulic conductivity and thickness K_a and m_a are set to $0.01\,\mathrm{m\ d^{-1}}$ and 30 m, respectively. The external hydraulic head H_a =100 m. Water is pumped from the only legal well LW_1 located in the node 28 at a daily rate of $10\,000\ \mathrm{m^3}$ (i.e. $\bar{q}$, the number of legal wells is equal to 1). Steady state conditions are assumed to prevail then.

At some time level *s*=0, two areas around nodes 34 and 44 are suspected to be subject of illegal pumping (i.e. $\bar{r}$, the number of potential illegal wells is 2). That is, a monitoring campaign was started in $\overline{m}=6$ monitoring wells MW_1 , MW_2 , …, MW_6 located in nodes 26, 36, 43, 25, 50 and 6, respectively, over a 9 day-period of observation that starts at $s_{\min}=7$ and ends at $s_{\max}=15$. Observed heads are obtained by launching the forward code from the initial distribution of Figure 2b. The flow domain is segmented into 97 triangular elements using *n* = 62 nodes.

In this paper, the above-presented methodology is checked against the following hupothetical illegal pumping scenario: $Q_{LW_1}^{(s)}=10000\,\mathrm{m^3d^{-1}}$ and $P_{PIW_1}^{(s)}=P_{PIW_2}^{(s)}=2500\,\mathrm{m^3d^{-1}}$. The test is based on uniform rates $P_{PIW_1}^{(s)}$ and $P_{PIW_2}^{(s)}$. Afterwards, the forward code is played in order to generate observed data set which is perturbed to 5%.

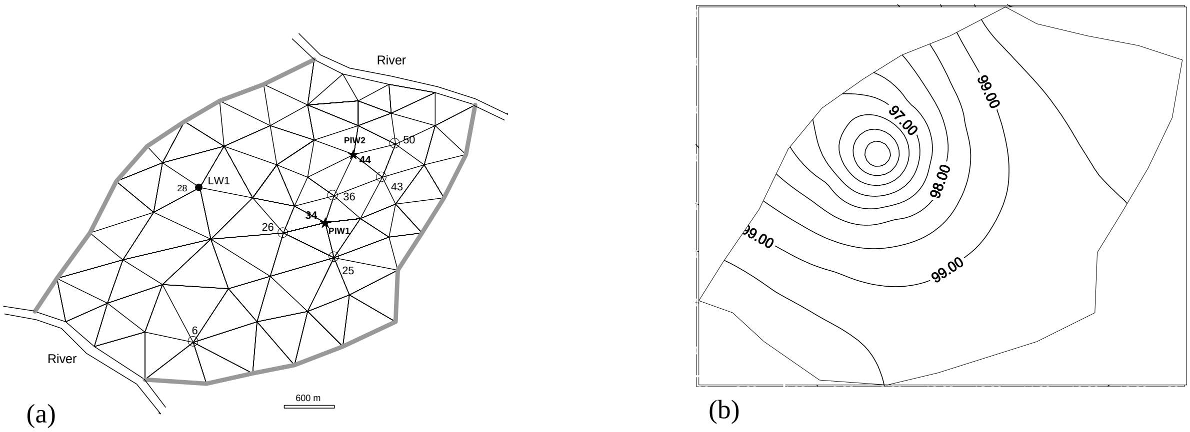

Figure 1 (a) Aquifer system configuration: black circle denotes the only legal well (LW_1); stars indicate potential illegal wells (*PIW*); unfilled circles show monitoring well (*MW*) position. (b) Initial hydraulic head distribution taken as the steady state solution to equation (1) for : $Q_{LW_1} = 10\,000\,\mathrm{m^3 d^{-1}};\ P_{PIW_1} = P_{PIW_2} = 0.$

	Days	Exact solution	Levenberg-Marquardt solution
	1	−2500	−2453.694
	2	−2500	−2570.460
	3	−2500	−2646.644
	4	−2500	−2636.510
	5	−2500	−2457.838
	6	−2500	−2448.028
	7	−2500	−2472.775
PIW_1	8	−2500	−2511.982
	9	−2500	−2484.737
	10	−2500	−2511.049
	11	−2500	−2480.410
	12	−2500	−2502.510
	13	−2500	−2498.170
	14	−2500	−2505.134
	15	−2500	−2490.996

Table 1 Identified pumping rates in $\mathrm{m^3 d^{-1}}$ for PIW_1

Under these conditions, Equation (4) is solved for ***P*** using Levenberg-Marquardt algorithm. As such, the procedure diverges: *J* assumes very small values while $\Delta\boldsymbol{P}$, the correction applied to ***P***, keeps large. A more rigorous analysis is carried out emphasizing one single iteration of the algorithm. Especially, the link between the correction $\Delta\boldsymbol{P}$ and the associated variation ΔJ of criterion *J* needs to be addressed. A regularizing procedure to improve the algorithm convergence has been implemented [8]. The results are presented on Table 1 for the potential illegal well PIW_1. Similar results are obtained for PIW_2.

It is shown that the procedure performs well within the whole period $[1, s_{\max}]$, even for the days *s* out of the observation window (i.e. $1 \leq s \leq 6$). The regularized pumping rates are not obscured by the added noise. An application including varying and non uniform pumping magnitudes is fully tretaed in [8].

5. CONCLUSION

This work formulates and solves the problem of illegal pumping in semi-confined aquifers. It aims at determining the optimal pumping scenario **P** that makes simulated and observed head distributions as close as possible. This is carried out iteratively using the output error criterion method that seeks to minimize a criterion measuring the misfit between observed and simulated heads. The methodology has been successfully applied to a constructed example of 2-D aquifer.

6. REFERENCES

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