Vol. **2**, **1**. 125-136 (2009)

Revue de Mécanique Appliquée et Théorique

Global Optimization for Structural Design

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Abstract
The present paper focus on the improvement of the efficiency of structural optimization, in typical structural optimization problems there may be many locally minimum configurations. For that reason, the application of a global method, which may escape from the locally minimum points, remain essential. In this paper, a new hybrid simulated annealing algorithm for large scale global optimization problems with constraints is proposed. We have developed a stochastic algorithm called SAPSPSA that uses Simulated Annealing algorithm (SA). In addition, the Simultaneous Perturbation Stochastic Approximation method (SPSA) is used to refine the solution. Commonly, structural analysis problems are constrained. For the reason that SPSA method involves penalizing constraints a penalty method is used to design a new method, called Penalty SPSA (PSPSA) method. The combination of both methods (Simulated Annealing algorithm and Penalty Simultaneous Perturbation Stochastic Approximation algorithm) provides a powerful hybrid stochastic optimization method (SAPSPSA), the proposed method is applicable for any problem where the topology of the structure is not fixed. It is simple and capable of handling problems subject to any number of constraints which may not be necessarily linear. Numerical results demonstrate the applicability, accuracy and efficiency of the suggested method for structural optimization. It is found that the best results are obtained by SAPSPSA compared to the results provided by the commercial software ANSYS.

Keywords Global optimization, simulated annealing, simultaneous perturbation stochastic approximation, hybrid method, structural optimization, FEM.

Résumé
Le présent article s'intéresse à l'amélioration de l'efficacité des méthodes d'optimisation des structures mécaniques. Dans les problèmes classiques d'optimisation structurale on peut trouver plusieurs configurations de minimums locaux. Pour cette raison, l'application d'une méthode d'optimisation globale, qui peut s'échapper des minimums locaux, devient essentielle. Dans cet article, un nouvel algorithme hybride pour les problèmes d'optimisation globale à grand échelle est proposé. Nous avons développé un algorithme stochastique nommé SAPSPSA qui utilise l'algorithme du recuit simulé (SA) hybridé avec la méthode d'approximation stochastique à perturbation simultanée (SPSA). Généralement, les problèmes d'analyse structurale sont des problèmes d'optimisation avec contraintes et comme la méthode SPSA s'applique aux problèmes sans contraintes, on a

utilisé alors la méthode de pénalité extérieure, ce qui nous a permis de développer la nouvelle méthode PSPSA. La combinaison des deux méthodes (SA et PSPSA) a fournit un algorithme hybride puissant et performant d'optimisation stochastique (SAPSPSA). La méthode proposée s'applique à n'importe quel problème d'optimisation où la topologie de la structure n'est pas fixe. Elle est simple et capable de traiter des problèmes sujets à n'importe quel nombre de contraintes non linéaires. Les résultats numériques démontrent l'applicabilité, l'exactitude et l'efficacité de la méthode suggérée pour l'optimisation des structures mécaniques. Les meilleurs résultats sont obtenus par SAPSPSA comparé aux résultats fournis par le logiciel commercial ANSYS.

Mots clés Optimisation globale, recuit simulé, approximation stochastique à perturbation simultanée, méthode hybride, optimisation structurale, MEF.

1. INTRODUCTION

The domain of design engineering is in constant progress and the race toward an optimal solution continues to good train. The increasing need for optimum structural designs with the most efficient use of material without violating constraints has given rise to several developments in the past three decades. A good deal of effort has been centred toward the search of optimal structural designs ([17], [10], [14], [23], [19], [27], [2], [7], [22], [16], [3]). In many of the previous studies the authors have used local search algorithms, such algorithms can only be successful if it is used to improve the current design ([17], [10], [14]) or only a small segment of the boundary is allowed to move ([17], [23], [19], [27], [10]), or only a small number of dimensional parameters are used to define its shape ([14], [2], [22], [16]) or if the objective function is convex. In fact, in real world's problems structures have became more and more complex, a design engineer working in the field of research and development has to often design completely new structures. The loading and support conditions of a particular design problem are usually known in advance, but the designer is unsure of what the final or optimal structure should look like. The essential goal of a designer in using an optimization algorithm is just to state the boundary conditions and let the algorithm do some iterations without human intervention until automatically produce the best design. In this respect, the previous studies had only a relative success. Many of them relied too much on designer's intuition including the choice of initial design, or imposed tight restrictions on the movements of boundary. For these reasons, global optimization should take part in structural problems. In some of the previous studies global search algorithms were used, but the accuracy remained questionable. Structural engineering and mathematical programming theory should, both of them, collaborate in some way to develop a powerful and sophisticated programming system for structural optimization, this includes the utilization of High Performance Finite Element solver as well as a robust global optimization method. In this paper, on the other hand, a new hybrid simulated annealing algorithm for global optimization with constraints is proposed. ASASPSA was designed in order to find the absolute minimum of an objective function without being sensitive to the starting point, capable of handing problems subject to any number of design variables or equality/inequality constraints, the solution is reached so that a reasonable compromise is made between the optimum and a shorter computational time. The present method can help researchers and practitioners devise optimal solutions to countless real-world problems. Numerical results demonstrate the applicability, accuracy and efficiency of the suggested method for structural optimization.

This paper is organized as follows. In Section 2 formal optimization problem is presented, in Section 3 the proposed method is described in details, Section 4 reports numerical experiments of the hybrid method through five benchmark functions well-known in the literature, the hybrid method is tested by five difficult nonlinear continuous functions and is compared with other global methods for performance analysis and in Section 5 some conclusions are derived.

2. PROBLEM STATEMENT

Formal optimization is associated with the specification of a mathematical objective function and a collection of factors (or parameters) that can be adjusted to optimize the objective function. In particular, one can formulate an optimization problem as follows:

$$\begin{cases} \min \ f(x) \\ Subject \ to \ \ x \in C \end{cases} \tag{1}$$

where $f : IR^p \rightarrow IR$ represents some loss function to be minimized, x represents the p-dimensional vector and $C \subset IR^p$ represents a constraint set defining the allowable values for the parameters x. In the present paper we are interested in problems for which x represents a vector of continuous parameters.

To solve problem (1) we have applied SAPSPSA, the method was designed to avoid being trapped by local optimum and to identify the global design optimum with high accuracy. The performance of our proposed method as a viable global optimization method is demonstrated by testing it on a number of benchmark functions with 2 - 30 dimensions. In addition, applicability of the algorithm on structural design was tested and successful results were obtained. Numerical as well as graphical results demonstrate the applicability, accuracy and efficiency of the proposed method.

3. PROBLEM SOLUTION

SAPSPSA is a hybridization of the two methods SA and PSPSA, the proposed method starts the search with the SA method until reaching the global solution then the founded solution is kept and the search is restarted with PSPSA to refine the solution. In the following the basic steps of SA and PSPSA methods are outlined, at the end of this section the algorithm of the proposed hybrid method SAPSPSA is presented.

3.1 Simulated annealing

The simulated annealing algorithm used in this paper is a simulated annealing Figure. 1 based on the work of Van Laarhoven and Aarts [24]. General steps and more details of basic simulated annealing algorithm can be found in [11] or Algorithm 1. In the following the main parts of the basic simulated annealing algorithm are outlined.

```
begin
Initialization
while not Termination do
      while maximal random perturbations not reached do.
         s_new := RAND (N(s))
         if s_new is better than s
                  s := s_new
         else
                  s := s_new with probability P (T; s; s_new)
         end if
      end while
Update T
end while
end
```

Algorithm 1. SA algorithm

3.1.1 Initialization
Generate initial solution s and initial parameters.

3.1.2 Termination
The algorithm terminates when T reaches the value of the minimal temperature allowed.

3.1.3 RAND (N(s))
Randomly choose a solution from N(s), where N(s) is the neighbourhood of s.

3.1.4 P (T; s; s_{new})
P (T; s; s_{new}) is the acceptance probability, which is defined as:

$$P(T; s; s_{new}) = \exp(-\Delta E / k_b.T),$$

where k_b is Boltzmann constant. $\Delta E = E(s_{new}) - E(s)$, is the amount of increase in the objective value caused by the uphill move and T is a parameter referred to as "annealing temperature".

3.1.5 Update T
Updating Temperature means cooling schedule. T_k temperature in the iteration k fulfils the following conditions:

$$T_k \geq 0 \text{ and } \lim_{k\to\infty} T_k = 0$$

The cooling schedule may be classic ([4], [11]) $T_k = T_0\ \alpha^k$, Logarithmic [13] $T_k = T_0 / \log(k+1)$ and so on. Where α is some constant such that $0 < \alpha < 1$, usually in the range of 0.90 - 0.99. T_0 = 0.01, 1, 100.

BSA Algorithm starts from an initial solution s at a high temperature T, and makes a series of moves according to RAND (N(s)). The change in the objective function values ΔE is computed at each

move. If the new solution results in decreased objective function value, it is accepted with probability 1. If the new solution yields increased objective function value, it is accepted with probability P (T; s; s_{new}). To avoid accepting large uphill move in the later stage of the search, the parameter T will be decreased over time by a schedule which is called "the cooling schedule".

3.2 Penalty Simultaneous Perturbation Stochastic Approximation

Commonly, structural problems are constrained. For the reason that SPSA method involves penalizing constraints a penalty method is used, the constrained problem is then converted into an unconstrained problem which allow as to design a new method for constrained optimization problems, called Penalty Simultaneous Perturbation Stochastic Approximation (PSPSA) method.

3.2.1 Penalty Method

Penalty method is a procedure for approximating constrained optimization problems by unconstrained ones. The approximation is accomplished in the case of penalty methods by adding to the objective function a term that prescribes a high cost for violation of the constraints. (For further details see [13]). Consider the problem

$$\begin{cases} Minimize \quad f(x), \quad x \in IR^n \\ \quad Subject\ to: \ x \in S \end{cases} \tag{2}$$

Where f is a continuous function on IR^n and S is a constraint set in IR^n. The idea of a penalty method is to replace problem (2) by an unconstrained problem of the form

$$Minimize \quad f(x) + \mu\, P(x). \tag{3}$$

where μ is a positive constant and P is a function on IR^n satisfying:

(i) P is continuous

(ii) $P(x) \geq 0$ for all $x \in IR^n$

(iii) $P(x) = 0$ if and only if $x \in S$

Suppose that

$$S = \{x : g(x) = 0\}$$

The problem (2) can be replaced by the unconstrained one as following:

$$\begin{cases} Minimize \quad f(x) + \mu\, g^2(x) \qquad \mu \ \ is\ large\ enough \\ \qquad\qquad\qquad\qquad x \in IR^n \end{cases}$$

Suppose now that S is defined by a number of inequality constraints

$$S = \{x : g_i(x) \leq 0, \quad i = 1,2,....., p\}$$

A very useful penalty function in this case is

$$P(x) = \sum_{i=1}^{p} (\max\ [0, g_i(x)])^2$$

For large μ it is evident that the minimum point of problem (3) will be in a region where P is small. Thus, for increasing μ it is expected that the corresponding solution points will approach the feasible

region S and, subject to being close, will minimize *f*. Ideally then, as $\mu \to \infty$ the solution point of the penalty problem will converge to a solution of the unconstrained problem.
More generally if the subset S is defined as

$$S = \{x : g_i(x) \le 0, \quad i = 1,2,.....,m \quad and \quad h_j(x) = 0, \quad j = 1,2,....., p\}.$$

Then the problem (2) is equivalent to that of (3) with a penalty function of the form:

$$P(x) = \sum_{i=1}^{m} (\max\,[0, g_i(x)])^2 + \sum_{j=1}^{p} \left|h_j(x)\right|^2$$

3.2.2 SPSA method

SPSA method is a local search based on a highly efficient and easily implemented simultaneous perturbation approximation to the gradient: this gradient approximation uses only two loss-function measurements, independent of the number of parameters being optimized. The following general algorithm of SPSA is based on the work of J. Spall ([20], [21]).

Step 1: Initialization and coefficient selection.
Set the SPSA gain sequences $a_k = a / (A+k)^{\alpha}$ and $c_k = c / k^{\gamma}$.
Step 2: Generation of the simultaneous perturbation vector.
Generate an *n*-dimensional random perturbation vector Δ_k where each of the *n* components of Δ_k is independently generated.
Step 3: Loss function evaluations.
Obtain two measurements of the loss function y (.) based on the simultaneous perturbation around the current $\hat{x}_k$: $y(\hat{x}_k + c_k\,\Delta_k)$ and $y(\hat{x}_k - c_k\,\Delta_k)$ with c_k and Δ_k from Steps 1 and 2.
Step 4: Gradient approximation.
Generate the simultaneous perturbation approximation to the unknown gradient $g(\hat{x}_k)$:

$$\hat{g}_k(\hat{x}_k) = \frac{y(\hat{x}_k + c_k\,\Delta_k) - y(\hat{x}_k - c_k\,\Delta_k)}{2\,c_k} \begin{bmatrix} \Delta_{k1}^{-1} \\ . \\ . \\ . \\ \Delta_{kn}^{-1} \end{bmatrix}$$

where Δ_{ki} is the i^{th} component of the Δ_k vector
Step 5: Updating *x* estimate.
Use the standard SA form:
$\hat{x}_{k+1} = \hat{x}_k - a_k\,\hat{g}_k(\hat{x}_k)$ to update $\hat{x}_k$ to a new value $\hat{x}_{k+1}$.
Step 6: Iteration or termination.
Return to Step 2 with k + 1 replacing k. Terminate the algorithm if there is little change in several successive iterates or the maximum allowable number of iterations has been reached.

SPSA Algorithm

Step 1 : Initialization and coefficient selection.
Initialize the starting point at random and the initial temperature T
Step 2 : Explore the search domain.
Use SA to avoid being trapped in local minima by making uphill move with the probability P (T; s; s_{new})
Step 3 : Exploit the search domain.
Keep the solution founded by SA and restart the search with PSPSA method in order to refine the solution
Step 4: Iteration or termination.
Terminate the algorithm if there is little change in several successive iterates or the maximum number of iterations has been reached.

SAPSPSA Algorithm

3.3 The proposed method

This article focuses on the enhancement of the efficiency of structural optimization by proposing the SAPSPSA method. In typical structural optimization problems there may be many locally minimum configurations. For that reason, the application of a global method, which may escape from the locally minimum points, remain essential. The proposed method has a good balance between exploration and exploitation. The new algorithm can be widely applied to a class of global optimization problems for continuous functions with box constraints. The main steps for SAPSPSA are as follows:

All the algorithms are coded in MATLAB 7.0 and run on PC, which has Intel 2.13 Ghz processor and 512 Mo RAM.

4. NUMERICAL EXAMPLES

4.1 Benchmark test functions

In this section, several tests have been performed on some well-known benchmark functions, see Appendix, with known global optima. To demonstrate the efficiency as well as the accuracy of the proposed method, five typical benchmark functions are used to test the convergence performance of the SAPSPSA algorithm. Each of them has more than one local optimum (see Table 1) and the behaviour of these benchmark functions varies; we have functions with a narrow valley such as Rosenbrock function, functions with many crowded local minima such as Shubert function and smooth functions such as De Joung function. As an example, the Shubert function processes about 760 local optimums (see the surface plot and the contour plot in Figs. 1 and 2).

Function (Name)	Search space	Number of local minima	The global optimum	f_{min}	Reference
DJ (De Jong)	$[-100, 100]^{n=30}$	1	$(0,0,...,0)^T$	0	[26]
SQ_n (Square)	$[-2, 4]^n$	1	$[(0,1,...,n-1)/n]^T$	1	[18]
CA (Camel)	$[-10, 10]^{n=2}$	6	$(\pm 0.089842, \mp 0.712656)^T$	-1.031628	[26], [9]
SHU (Shubert)	$[-10, 10]^{n=2}$	760	$(-1.42513,-0.80032)^T$	-186.730909	[8], [9]
RO (Rosenbrock)	$[-10, 10]^{n=2}$	unavailable	$(1,1)^T$	0	[12]

Table 1 Basic information about 5 benchmark functions

In order to examine the capability of SAPSPSA in function minimization, a comparison is made with various prominent algorithms from the literature (see Table 2). Solutions obtained are then compared with these algorithms (see Table 3).

As shown in Table 3, the SAPSPSA can converge fairly close to the global optimum (verified by the theoretical optima). We also notice that, when compared with other methods SAPSPSA demonstrate its accuracy in achieving the global solution; the new proposed method produces the global optimal value with higher accuracy, compared to that of SA, higher accuracy ensures the degree of consistency in producing the global optimal value and hence the proposed method shows better consistency than the basic simulated annealing. We can claim that the hybridization of the two methods namely, SA and SPSA enhances the robustness of the basic SA.

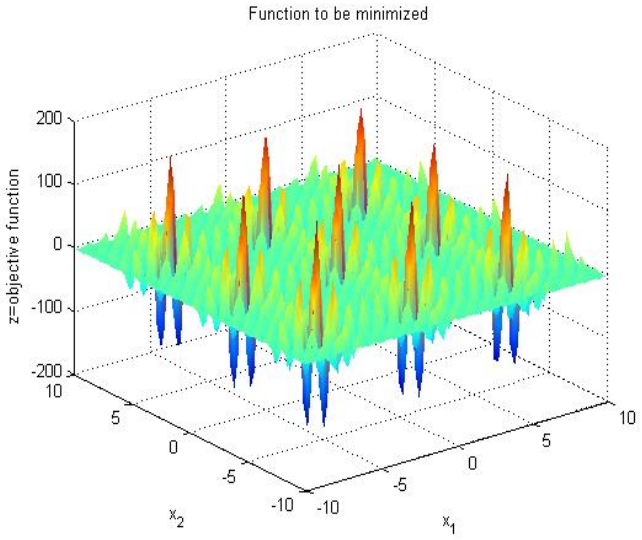

Figure. 1 The surface plot of Shubert function

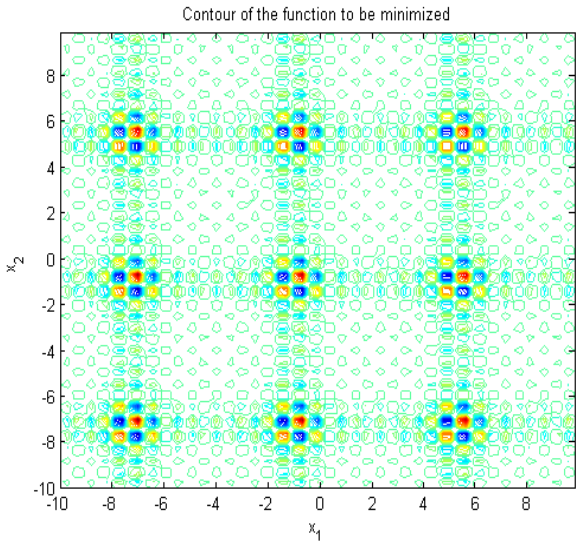

Figure. 2 The contour plot of Shubert function

Method	Name	Reference
NFF	New Filled Function	[25]
ePSO	extrapolation Particle Swarm Optimization	[1]
ODE	Orthogonal based Differential Evolution	[5]
ARSET	Adaptive Random SEarch Technique	[6]
RSM	Response Surface Methodology	[18]
CGVNS	Continuous General Variable Neighborhood Search	[15]
HS	Harmony Search	[12]
SA	Simulated Annealing	This paper
SAPSPSA	Simulated Annealing Penalty SPSA	This paper

Table 2 Global optimization methods used for performance analysis

Test functions	Comparison of results			Accuracy	
	Number of variables	SAPSPSA	Other methods	SA	SAPSPSA
SQ_3	3	1	RSM: 1.021	$3.3231.e^{-004}$	0
SQ_8	8	1	RSM: 1.119	$9.4220.e^{-002}$	0
SHU	2	-186.7309088	NFF: -186.7309088	$4.5620e^{-006}$	$9.0492e^{-010}$
DJ	30	$5.1622e^{-036}$	ODE: $2.06e^{-023}$	$5.3948.e-^{001}$	$5.1622e^{-036}$
CA	2	-1.03162841	ePSO: -1.031604	$1.8822e^{-005}$	0
RO	2	$1.61e^{-018}$	HS: $5.68e^{-010}$ ARSET: $4.02e^{-016}$	$3.6699e^{-007}$	$1.61e^{-018}$

Table 3 Performance of SAPSPSA on 5 test functions

4.2 Structural design

The design problem considered is a plate shape, shown in Fig 3, such that the maximum equivalent stress reaches a minimum value with the volume as a constraint.

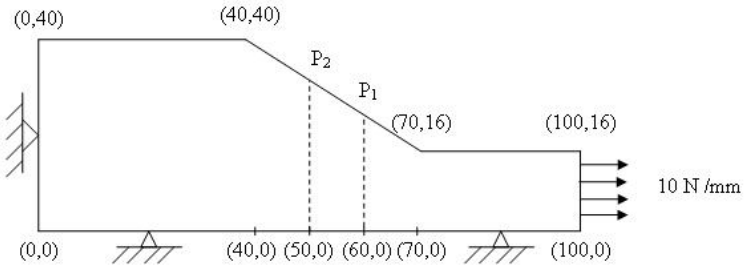

Figure 3. Example of a plate.

The objective of this problem is to find a feasible set of dimensions P_1 and P_2 (denoted by $x = [x_1; x_2]$) that minimize the Von Mises stress without violating constraints on the volume. The optimization problem is formulated as follows:

$$\begin{cases} \text{Minimize } \sigma_{von} \\ \text{Subject to}: V \le V_0 \\ 16 \le DV_i \le 40 \qquad i = 1, 2 \end{cases}$$

where σ_{von} is the Von Mises stress, V is the volume, V_0 is the initial volume obtained with initial design variables, i is the number of design variables and DV_i is the design variable number i. In this case, we have two design variables P_1 and P_2 as shown in Fig. 3.

The purpose of this study is to show the applicability, the efficiency and the consistency of the proposed method. For these reasons a comparison is made with the international software ANSYS. The optimization method used for ANSYS is First Order method (FO). Table 4 shows the plate results when using ANSYS and our method. The optimal Von Mises stress obtained by our method is less then that obtained by the FO method. This way, the Von Mises stress value reduction is **24.29 %** from initial shape (see Fig 4 and 5), witch leads to economic structure.

Optimal design variables (x) and objective function value (f(x))	Methods			Reduction (%)
	SAPSPSA	FO	Initial shape	
DV_1 (mm)	17.5748	17.9990	24	
DV_2 (mm)	22.0407	22.9110	32	**24.29**
σ_{von} (N/mm)	11.5640	12.0345	15.2745	

Table 4 Numerical Results for the plate problem

As evidenced by the computational results given here, the proposed hybrid method shows a great deal of promise as an optimization method with higher accuracy, efficiency and robustness in solution quality for structural problems.

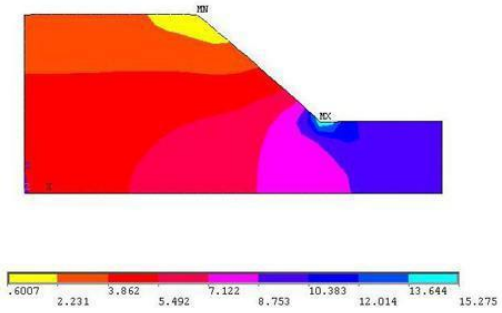

Figure 4 : The Graph of the Initial Model

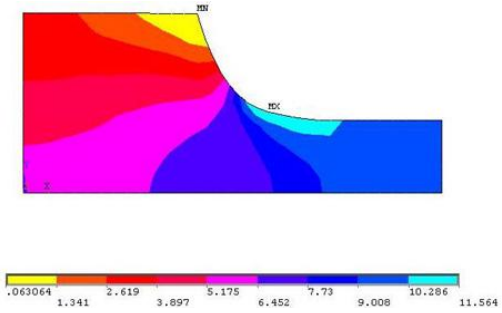

Figure 5 : The Graph of the Optimized Model

5. CONCLUSION

In this paper a new global hybrid method ASASPSA for large scale problems is proposed. The new algorithm can be widely applied to a class of global optimization problems for continuous functions

with box constraints. The experimental results show that the present method has proved the robustness and high performance of its algorithm. We can clearly see from numerical and graphical results that the algorithm can yield the global optimum with high accuracy.

6. APPENDIX

$$SQ_n = \left[\sum_{i=1}^{n} \left(x_i - \frac{i-1}{n} \right)^2 \right] + 1$$

$$SHU = \left\{ \sum_{i=1}^{5} i \cos\left[(i+1)\, x_1 + i \right] \right\} \left\{ \sum_{i=1}^{5} i \cos\left[(i+1)\, x_2 + i \right] \right\}$$

$$DJ = \sum_{i=1}^{n} x_i^2$$

$$CA = 4\, x_1^2 - 2.1\, x_1^4 + \frac{1}{3} x_1^6 + x_1\, x_2 - 4\, x_2^2 + 4\, x_2^4$$

$$RO = 100\, \left(x_2 - x_1^2 \right)^2 + \left(1 - x_1 \right)^2$$

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