

Multi-variant theory for the behaviour of single crystal shape memory alloys

M. O. Bensalah

Laboratoire de Mécanique et des Matériaux Département de physique,
Faculté des Sciences de Rabat, B.P. 1014, Rabat, Maroc.

H. Chakchak

Laboratoire de Mécanique et des Matériaux Département de physique,
Faculté des Sciences de Rabat, B.P. 1014, Rabat, Maroc.

A. Ghazali

Laboratoire de Mécanique et des Matériaux Département de physique,
Faculté des Sciences de Rabat, B.P. 1014, Rabat, Maroc.

Abstract:

In this work, we study the shape memory alloys (SMA) micromechanical behaviour in the perfect transformation plasticity. This study is based on the kinematical description of the physical mechanisms at the origin of the phenomenon, and on the thermodynamical analysis of the irreversible processes presented by these alloys in such a transformation. The pseudoelastical behaviour of SMA is described by defining a pseudoelastical potential through the Gibbs theory, and by application of the second principle of the thermodynamic. This approach allowed us to incorporate the change of the martensite variants shape in such thermomechanical behaviour and to analyse the hysteretic behaviour of these materials.

Résumé

Dans ce travail, on étudie le comportement thermomécanique des alliages à mémoire de forme AMF en plasticité de transformation pure. L'étude est basée sur une description cinématique de la mécanique physique origine du phénomène et une analyse thermodynamique du processus irréversible présenté par ces alliages dans la transformation. Le comportement pseudo-élastique des AMF est décrit par la définition d'un pseudo-potential à partir de la théorie de Gibbs et l'application du second principe de la thermodynamique. Cette approche introduit le changement des variantes de martensite dans le comportement thermomécanique et une analyse du comportement hystérique de ces matériaux.

1. INTRODUCTION

The shape memory alloys (SMA) undergoing a thermoelastic martensitic transformation, which could be induced either by variation of temperature or by application of a sufficient stresses [1,2], have received a considerable attention in the last three decades because of their particular shape memory effect (SME) and pseudoelastical behaviours. The intensive investigations have allowed the understanding of the physical mechanisms of these alloys, during transformation, in order to exploit them in a large field of industrial applications. Hence, the interest of establishing some modellings which are able to describe the principal characteristics of the thermomechanical behaviour of SMA

and to satisfy the engineers' requirements. Since the eighties, some attempts have been made from a thermomechanics point of view to describe the transformation pseudoelasticity and the SME: Frémond [3] gave a macroscopic thermodynamic description of SMA founded on the generalised standard materials theory. To take into account the physical constraints on volume fractions, this model introduces the formalism of the indicators. However, this description neglects the interaction between phases.

Müller et al. [4] carried out a macroscopic thermodynamic description of single crystals behaviour from the free energy associated to the phase transformation of first order of Van Der-Waals type. To take into account the hysteresis, an interfacial energy, proportional to the product of volume fractions of both phases, has been added.

Raniecki et al. [5] developed a macroscopic thermodynamic model associated to solid phase change under the imposed conditions by the thermodynamics of irreversible processes. The transformation is, hence, controlled by the generalised thermodynamic forces. The hysteresis is described using a thermodynamical analysis of the interaction energy between parent and product phases. But this approach gives no interpretation for the change of the hysteresis feature with loading conditions.

Falk [6] described the pseudoelastical behaviour of SMA by adopting a micro-macro approach. Polycrystal behaviour is obtained from the description of the single crystal behaviour. The polycrystal is considered as being constituted of grains of given crystallographic orientations and possessing an identical behaviours. The behaviour is determined by the average operation by using the Voigt and Reuss hypothesis.

Patoor et al. [7,8] developed a model, which is essentially a micro-macro approach by using a self-consistent scheme (identification of elementary mechanisms and integration of these phenomenons to the macroscopic scale). This model proves out to be extremely attractive, however its reversible character doesn't allow describing the hysteretic cycles.

Sun et al. [9] Introduced a micromechanical description of the transformation plasticity into the continuum of the macroscopic behaviour. But the size of the constitutive element was too large for an adequate description of the strain mechanisms.

In this study, we limit ourselves to the pure transformation plasticity and to the formation and development of the martensite from the austenite, in the case of a pseudoelastical behaviour. For this, we present firstly a kinematic study associated to such a transformation in small deformations, where the macroscopic behaviour is obtained using the micro-macro approach. The pseudoelastical behaviour is described by defining a pseudoelastical potential, through the Gibbs theory, and by applying the second thermodynamic principle. This allows us to incorporate the contribution of the change of martensite shape (displacement of the austenite-martensite interfaces and martensite-martensite inter-variant boundaries) and to describe the martensite variants reorientation in such behaviour. This approach allowed us to study the hysteretic behaviour of these alloys.

2. KINEMATIC OF THE TRANSFORMATION

The choice of the used description is an important stage of the modelling. In order to consider the structure of a single crystal and the existence of twenty-four martensite variants crystallographically equivalents [10, 11], one adopts, as a description scale, the martensite variant. One considers, therefore, as element of basis for the material global behaviour modelling, the transformation strain associated to the formation of a martensite variant h in the single crystal. This transformation strain, noted ε^h , is a function of only the strain amplitude g and the orientation R^h of the considered variant [7];

$$\varepsilon_{ij}^h = g R_{ij}^h \quad (1)$$

$$R_{ij}^h = \frac{1}{2}(n_i^h m_j^h + n_j^h m_i^h) \quad (2)$$

where \vec{n}^h is the normal of the variant h and \vec{m}^h is the transformation direction.

We consider an homogeneous and elastic material of V volume undergoing a phase transformation, the macroscopic transformation strain E^{Tr} is the volume average of the local transformation strain $\varepsilon^{Tr}(\mathbf{r})$,

$$E_{ij}^{Tr} = \frac{1}{V} \int_V \varepsilon_{ij}^{Tr}(\mathbf{r}) dv \quad (3)$$

$$E_{ij}^{Tr} = \frac{1}{V} \int_V \sum_h \varepsilon_{ij}^h \Theta^h(\mathbf{r}) dv \quad ; \quad \text{where} \quad \Theta^h(\mathbf{r}) = \begin{cases} 0 & \text{si } \mathbf{r} \notin v^h \\ 1 & \text{si } \mathbf{r} \in v^h \end{cases} \quad (4)$$

$$E_{ij}^{Tr} = \sum_h \varepsilon_{ij}^h f^h \quad (5)$$

$f^h (= V^h/V)$ is the volume fraction of the variant h .

3. PSEUDOELASTICAL POTENTIAL OF AUSTENITE MARTENSITE TRANSFORMATION

In order to describe the SMA pseudoelastic behaviour, we adopt a thermodynamic potential through the variation of Helmholtz's free energy Φ associated to the thermoelastic martensitic transformation. This energy is defined, in a unit volume, as the sum of the variation of chemical free energy ΔG_{ch} , of the interfacial free energy W_{sur} associated to the creation of the interfaces during the transformation and of the elastic deformation energy (mechanical energy) W [12]. In the stage of propagation, the observed elongation shape of the martensite plates shows that W_{sur} is negligible compared to the mechanical energy; this contribution will be neglected in the following of this study. Φ is thus written:

$$\Phi(E, T, y_k) = \Delta G_{ch} + W \quad (6)$$

This energy depends on the control variables (the total strain E and the temperature T) and on the internal variables y_k . The kinematic study showed that the volume fractions of the different martensite variants could correctly describe the microstructural state of the material. We choose, therefore, these parameters as internal variables.

The variation of Gibbs's free energy Ψ is best adapted to the description of the SMA thermomechanical behaviour because the martensitic transformation can be induced by a drop in temperature or by applying a stress Σ :

$$\Psi(\Sigma_{ij}, T, f^h) = \Sigma_{ij} E_{ij} - \Phi(E_{ij}, T, f^h) \quad (7)$$

The variation of chemical free energy, ΔG_{ch} , per unit of volume, associated to the formation of martensite from an austenite state is well represented by a linear function of the difference between the current temperature and the equilibrium transformation temperature T_0 . ΔG_{ch} is proportional of the total amount f of martensite formed in the unit volume [12]:

$$\Delta G_{ch} = B(T - T_0) f \quad (8)$$

B is a coefficient of proportionality.

The mechanical energy W is made up of two terms, which have different physical origins. On the one hand the elastic energy of deformation W^e caused by the applied macroscopic stress field Σ :

$$W^e = \frac{1}{2} \Sigma_{ij} E_{ij}^e \quad (9)$$

On the other hand, the stored energy W_b associated to the internal stresses field $\sigma(r)$ generated by the incompatibilities of the transformation strain:

$$W_b = -\frac{1}{2V} \int_V \sigma_{ij}^{\text{Tr}}(r) \varepsilon_{ij}^{\text{Tr}}(r) dv \quad (10)$$

Considering the martensite variant h as ellipsoidal inclusion characterised by the Eshelby's tensor S^h , the expression of the internal stress in the variant h (σ^h) is given by [13]:

$$\sigma_{ij}^h = C_{ijkl} (I_{klpq} - S_{klpq}^h) (E_{pq}^{\text{Tr}} - \varepsilon_{pq}^h) \quad (11)$$

Where C is the elasticity tensor and I is the identity fourth order tensor.

It is important to note that the relationship (11) is, as reported by many authors (Sabar [14]; Patoor [8,15]; Sun [9]; Agouram [16]), an approximate way to determine the internal stresses.

The stored energy W_b becomes:

$$W_b = -\frac{1}{2} \sum_h C_{ijkl} (I_{klpq} - S_{klpq}^h) (E_{pq}^{\text{Tr}} - \varepsilon_{pq}^h) \varepsilon_{ij}^h f^h \quad (12)$$

From equations (8), (9) and (12) the pseudoelastical potential Ψ is written as:

$$\begin{aligned} \Psi(\Sigma_{ij}, T, f^h) = & \frac{1}{2} \Sigma_{ij} E_{ij}^e + \frac{1}{2} \sum_h C_{ijkl} (I_{klpq} - S_{klpq}^h) (E_{pq}^{\text{Tr}} - \varepsilon_{pq}^h) \varepsilon_{ij}^h f^h \\ & - B(T - T_0) \sum_h f^h + \Sigma_{ij} \sum_h \varepsilon_{ij}^h f^h \end{aligned} \quad (13)$$

The pseudoelastical potential Ψ describes the austenite-martensite system state in terms of the applied stress Σ , the temperature T , the active martensite variants volume fractions f^h (internal variables), the morphologies of the latter as well as their orientations.

It is an obvious experimental fact, for the SMA, that the (austenite \leftrightarrow martensite) forward and reverse transformation ways are not the same. This is owing to a dissipated energy during these transformations. In the following sections, we model this energy and we describe the pseudoelastical behaviour of these alloys.

4. ENERGY DISSIPATION ASSOCIATED TO THE TRANSFORMATION

Shape Memory alloys present in their thermomechanical behaviour an hysteresis on the stress-strain and strain-temperature curves in the course of load-unload and cooling-heating cycles. This particularity of behaviour, which is very important essentially in the tiredness tests, is partly attributed to the intrinsic phenomena of dissipation related to the transformation [3,17], such as internal friction between austenite-martensite and martensite-martensite interfaces, production of defects and temperature variation accompanying the change of austenite-martensite phase [18]. We model this phenomenon, according to experimental data [19], by choosing the energy dissipation W_{diss} as:

$$W_{diss} = D_1 f + D_2 f^2 \quad , \quad f = \sum_h f^h \quad (14)$$

In what follows, we determine the constant D_1 and we discuss the influence of the constant D_2 on the thermomechanical behaviour of the SMA.

5. SMA THERMOMECHANICAL BEHAVIOUR

In the course of transformation, one of the two phases develops to the detriment of the other by displacement of the interface separating the two phases [14, 20, and 15]. Experimental observations on SMA in transformation show that the change of shape of the martensite variants is described by the displacement of their boundaries [21, 22, and 23]. This variation of shape is expressed explicitly by the variation of Eshelby's tensor S^h associated to each variant h . We consider this tensor as a parameter (in addition to f^h , T and Σ) of potential Ψ .

In the presence of dissipation phenomena, supposing that for each volume V^h of ellipsoidal shape, the transformation strain ε^h is constant in time and uniform in space, the second principle of the thermodynamic requires [9,15]:

$$\dot{\Psi}_{|\Sigma, T} = \frac{\partial \Psi}{\partial f^h} \dot{f}^h + \frac{\partial \Psi}{\partial S^h} \dot{S}^h \geq 0 \quad (15)$$

And

$$\dot{\Psi}_{|\Sigma, T} = \dot{W}_{diss} \quad (16)$$

Where

$$\dot{\Psi}_{|\Sigma, T} = \sum_{ij} \frac{\partial E_{ij}^{Tr}}{\partial f^h} \dot{f}^h - \frac{\partial \Delta G}{\partial f^h} \dot{f}^h - \dot{W}_b \quad (17)$$

- The rate of the W_b :

We consider the residual volume of austenite as a 25th variant of volume fraction f^{25} [16]. We can write:

$$\sum_n^{25} f^n = 1 \quad (18)$$

Taking into account this relation, the equation (12) becomes:

$$W_b = -\frac{1}{2} \sum_h C_{ijkl} (I_{klpq} - S_{klpq}^h) (E_{pq}^{Tr} - \varepsilon_{pq}^h \sum_n^{25} f^n) \varepsilon_{ij}^h f^h \quad (19)$$

The rate of W_b is then:

$$\begin{aligned} \dot{W}_b &= \frac{1}{2} C_{ijkl} [(I_{klpq} - S_{klpq}^h) (\dot{I} - \dot{f}^h) \varepsilon_{pq}^h \varepsilon_{ij}^h \\ &\quad - \sum_{k \neq h} \{ (I_{klpq} - S_{klpq}^h) \varepsilon_{pq}^k \varepsilon_{ij}^h + (I_{klpq} - S_{klpq}^k) (\varepsilon_{pq}^h - \varepsilon_{pq}^k) \delta_{ij}^k \} f^k] \dot{f}^h \\ &\quad + \frac{1}{2} C_{ijkl} \dot{S}_{klpq}^h (E_{pq}^{Tr} - \varepsilon_{pq}^h) \varepsilon_{ij}^h f^h \end{aligned} \quad (20)$$

The Eshelby's tensor S^h (describing the shape of the martensite variants) depends only of half-axes of the ellipsoid $a_1(t)$, $a_2(t)$ and $a_3(t)$, \dot{S}^h is written as follows:

$$\dot{S}^h = \sum_{\alpha=1}^3 \frac{\partial S^h}{\partial a_\alpha} \dot{a}_\alpha = \sum_{\alpha=1}^3 \frac{\partial S^h}{\partial a_\alpha} \frac{\partial a_\alpha}{\partial f^h} \dot{f}^h \quad (21)$$

The shape of martensite variants is ellipsoidal, the last equation can be written as:

$$\dot{S}^h = \sum_{\alpha=1}^3 \frac{a_\alpha}{f^h} \frac{\partial S^h}{\partial a_\alpha} \dot{f}^h \quad (22)$$

From equations (5), (8), (20) and (22) $\dot{\Psi}_{|\Sigma, T}$ is then:

$$\begin{aligned} \dot{\Psi}_{|\Sigma, T} &= \left\{ \Sigma_{ij} \varepsilon_{ij}^h - B(T - T_0) - \frac{1}{2} C_{ijkl} [(I_{klpq} - S_{klpq}^h) (\dot{I} - \dot{f}^h) \varepsilon_{pq}^h \varepsilon_{ij}^h \right. \\ &\quad - \sum_{k \neq h} \{ (I_{klpq} - S_{klpq}^h) \varepsilon_{pq}^k \varepsilon_{ij}^h + (I_{klpq} - S_{klpq}^k) (\varepsilon_{pq}^h - \varepsilon_{pq}^k) \delta_{ij}^k \} f^k \\ &\quad \left. + \sum_{\alpha=1}^3 a_\alpha \frac{\partial S_{klpq}^h}{\partial a_\alpha} (E_{pq}^{Tr} - \varepsilon_{pq}^h) \varepsilon_{ij}^h \right] \dot{f}^h \end{aligned} \quad (23)$$

From equations (14), (16) and (23), We obtain the single crystal behaviour law in the pure transformation plasticity:

$$\begin{aligned} \Sigma_{ij} R_{ij}^h = & \frac{B}{g} (T - T_0) + \frac{D_1}{g} + \frac{2D_2}{g} \sum_h f^h + \frac{1}{2} g C_{ijkl} \left[(I_{klpq} - S_{klpq}^h) (1 - f^h) R_{pq}^h R_{ij}^h \right. \\ & - \sum_{k \neq h} \left\{ (I_{klpq} - S_{klpq}^h) R_{pq}^k R_{ij}^h + (I_{klpq} - S_{klpq}^k) (R_{pq}^h - R_{pq}^k) R_{ij}^k \right\} f^k \\ & \left. + \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} \left(\sum_k R_{pq}^k f^k - R_{pq}^h \right) R_{ij}^h \right] \end{aligned} \quad (24)$$

This is written as follows:

$$\tau^h = \tau^0 + H^{hh} f^h + \sum_{k \neq h} H^{hk} f^k \quad (25)$$

With τ^h the resolved stress applied on the variant h ,

$$\tau^h = \Sigma_{ij} R_{ij}^h \quad (26)$$

τ^0 Is the initial resolved stress

$$\tau^0 = \frac{B}{g} (T - T_0) + \frac{D_1}{g} + \frac{1}{2} g C_{ijkl} (I_{klpq} - S_{klpq}^h) R_{pq}^h R_{ij}^h - \frac{1}{2} g C_{ijkl} \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} R_{pq}^h R_{ij}^h \quad (27)$$

H^{hh} is the own energy of variants,

$$H^{hh} = \frac{2D_2}{g} - \frac{1}{2} g C_{ijkl} \left[(I_{klpq} - S_{klpq}^h) R_{pq}^h R_{ij}^h - \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} R_{pq}^h R_{ij}^h \right] \quad (28)$$

And H^{hk} is the term describing the interactions as well as reorientations between variants,

$$H^{hk} = \frac{2D_2}{g} - \frac{1}{2} g C_{ijkl} \left[\left((I_{klpq} - S_{klpq}^h) - \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} \right) R_{pq}^k R_{ij}^h + (I_{klpq} - S_{klpq}^k) (R_{pq}^h - R_{pq}^k) R_{ij}^k \right] \quad (29)$$

The constant D_1 in the equation (24) could be determined, in order to describe the forward (reverse) transformation, by cooling the austenite (heating the martensite) at $\Sigma_{ij}=0$ until $T=M_s$, M_s the martensite start temperature on cooling, (until $T=A_f$, A_f the austenite finish temperature on heating from the martensite), in this case $f^h=0$. We obtain the laws describing respectively the forward and reverse transformations:

$$(\tau^h)_c = \tau_c^0 + H^{hh} f^h + \sum_{k \neq h} H^{hk} f^k \quad (30)$$

Respectively

$$(\tau^h)_d = \tau_d^0 + H^{hh} f^h + \sum_{k \neq h} H^{hk} f^k \quad (31)$$

Where $(\tau^h)_c$ and $(\tau^h)_d$ respectively the resolved are stresses applied on variant h during forward and reverse transformations:

$$(\tau^h)_c = \Sigma_{ij}^c R_{ij}^h, \quad (\tau^h)_d = \Sigma_{ij}^d R_{ij}^h \quad (32)$$

τ_c^0 and τ_d^0 Are respectively critical stresses of forward and reverse transformations

$$\tau_c^0 = \frac{B}{g}(T - M_s) + \frac{1}{2} g C_{ijkl} (I_{klpq} - S_{klpq}^h) R_{pq}^h R_{ij}^h - \frac{1}{2} g C_{ijkl} \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} R_{pq}^h R_{ij}^h \quad (33)$$

$$\tau_d^0 = \frac{B}{g}(T - A_f) + \frac{1}{2} g C_{ijkl} (I_{klpq} - S_{klpq}^h) R_{pq}^h R_{ij}^h - \frac{1}{2} g C_{ijkl} \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} R_{pq}^h R_{ij}^h \quad (34)$$

Hysteretic thickness of both stress (at constant imposed temperature) and temperature (at constant imposed stress) is obtained by the difference between the equations (30) and (31):

$$\Delta \Sigma_{ij} R_{ij}^h = \frac{B}{g} (A_f - M_s) \quad \text{where} \quad \Delta \Sigma_{ij} = \Sigma_{ij}^c - \Sigma_{ij}^d \quad (35)$$

And

$$T^{co} - T^{he} = \Delta T = (M_s - A_f) \quad (36)$$

Where T^{co} and T^{he} are respectively the cooling and heating temperatures

6. RESULTS AND DISCUSSIONS

Numerical results, for the single crystal behaviour obtained by this approach, are represented in this section. Computations are applied to a Cu-19.0 Zn-7.0 Al (wt %) SMA. Crystallographical characteristics of martensitic transformation used in this study are derived from De Vos et al. [24]. A comparison is realised between obtained results and experimental measurements performed on this kind of alloy by Fu et al. [25] and Huo et al. [26].

The choice of this alloy contains multiple advantages [26, 27]:

- These materials don't present any coupling between the classical plasticity and the transformation plasticity,
- For the pseudoelastical behaviour and cooling-heating test under uniaxial tension, the transformation is produced according to one variant,
- During transformation, the volume variation is negligible (in comparison to transformation strain).

We assume that, among the twenty-four variants of martensite, only a particular variant which is energetically most favourable can be preferentially formed and grow up to the single crystal boundary. The equation (25), becomes:

$$\tau^h = \tau^0 + H^{hh} f^h \tag{37}$$

In the beginning of transformation, at the resolved stress threshold $(\tau^h)_s, f^h = 0$ the last equation becomes:

$$(\tau^h)_s = \tau^0 \tag{38}$$

And in the end of transformation, at the final resolved stress $(\tau^h)_f, f^h = 1$ we have:

$$(\tau^h)_f = \tau^0 + H^{hh} \tag{39}$$

The difference between the final and threshold resolved stresses is positive, this is expressed as:

$$H^{hh} > 0 \quad \text{ie} \quad \left[\frac{2D_2}{g} - \frac{1}{2} g C_{ijkl} \left((I_{klpq} - S_{klpq}^h) - \sum_{\alpha=1}^3 a_{\alpha} \frac{\partial S_{klpq}^h}{\partial a_{\alpha}} \right) R_{pq}^h R_{ij}^h \right] > 0 \tag{40}$$

In CuZnAl SMA De Vos et al. pointed out that the habit plane normal and transformation direction are of (2-11-12) type. We assume that the elastic behaviour of two phases is identical, isotropic and homogenous. Transformation displacement g is about 0.23, the coefficient B is equal to 0.23 MPa/K and the Poisson's ratio is about 0.33 [8]. The martensite variants shape are assumed to be ellipsoidal flattened « penny shape » of half axes $a_1 = a_2 = 100a_3$.

At a given values of Σ and T , from equation (37) for both forward and reverse transformations, we have a linear equations of unknown f^h . By computing f^h , using equation (5), the problem is completely solved.

Pseudoelastic behaviour of single crystal is computed in that way, using experimental data from Refs. [25-26]. the tensile tests are simulated and compared to the experimental results for different temperatures (figure 1).

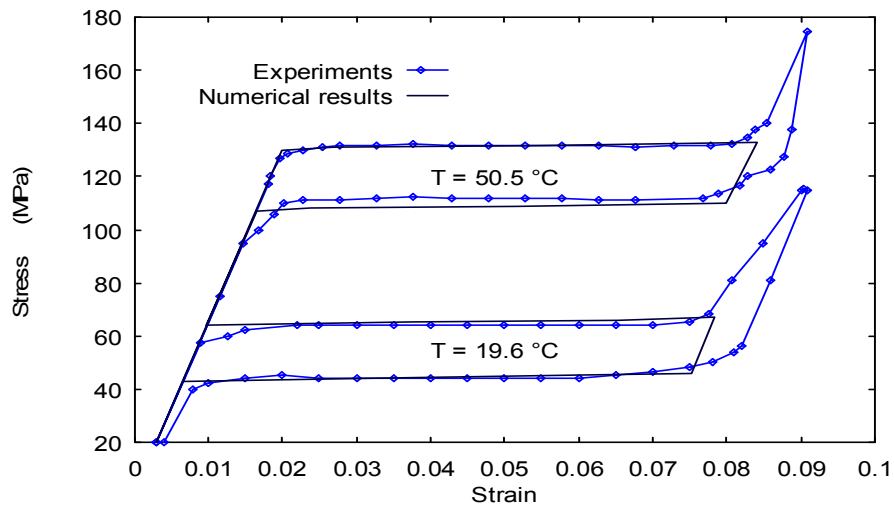


Figure 1: Pseudoelastic behaviour of the single crystal in uniaxial tension.

The size of the hysteresis loop, which is independent of temperature T , is correctly described and the apparent critical transformation stress fits with experimental one. When the temperature of the experimental test is increased from 19.6°C to 50.5°C, numerical results remain in good accordance. The transformation produces with an identical slope in the forward and reverse transformation. This is owing to the role played by the internal stresses of incompatibilities and by the material microstructure in transformation (orientation, shape and mobility of the interfaces).

At a temperature $T=19.6^\circ\text{C}$, simulations made on single crystals having different orientations are illustrated in figure 2. We observe that the transformation start stress and the transformation strain at saturation depend on the single crystal orientation.

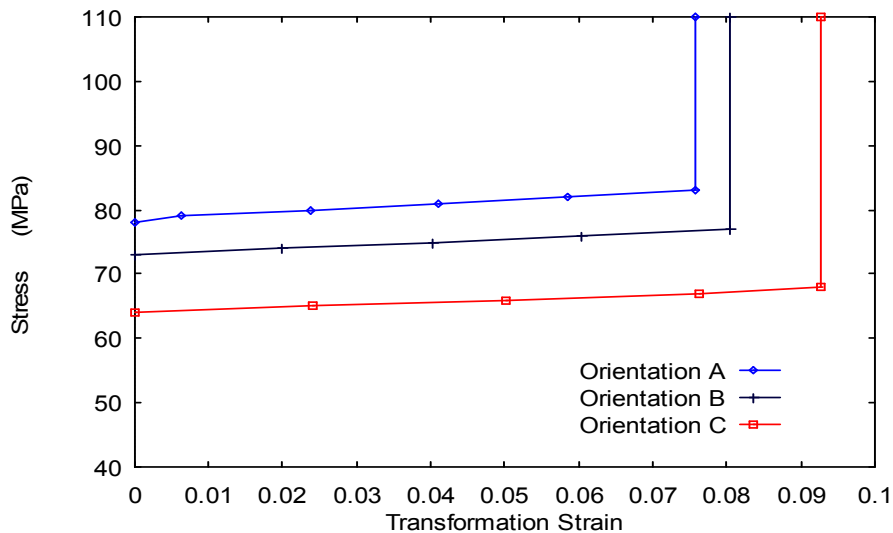


Figure 2: Influence of the crystallographic orientation on the behaviour of the single crystals CuZnAl.

Orientation A: $\varphi^1 = 207.800^\circ$, $\varphi = 74.350^\circ$, $\varphi^2 = 231.970^\circ$

Orientation B: $\varphi^1 = 354.730^\circ$, $\varphi = 88.910^\circ$, $\varphi^2 = 229.610^\circ$

Orientation C: $\varphi^1 = 54.490^\circ$, $\varphi = 76.140^\circ$, $\varphi^2 = 69.660^\circ$

Many experimental works have established that the transformation does not proceed in a symmetrical way in the stress space. Vacher [28] have observed a very large difference between the mechanical response for tensile and compressive tests. This result was confirmed by Roumagnac [29] and Rogued [30]. The micromechanical approach developed in this paper successfully deals with this non-symmetrical aspect of the pseudoelastic behaviour (figure 3). This dissymetry, accompanied of a variation of the start stress and of a reduction of the final transformation strain, is related to the crystallographic effect in which the transformation is produced by activating different martensite variants for tensile and compressive tests.

In the case of the thermal tests under constant imposed uniaxial tension stresses, computational results obtained by this way are in very good agreement with experiments (figure 4). The width of the hysteresis is correct and the characteristic temperatures of transformation, contrarily to the results obtained by Falk [6], are not confused. This is owing to the role played by the internal stresses that have been taken into account in this approach. The cooling-heating curves show that the macroscopic transformation strain depends on the imposed stress.

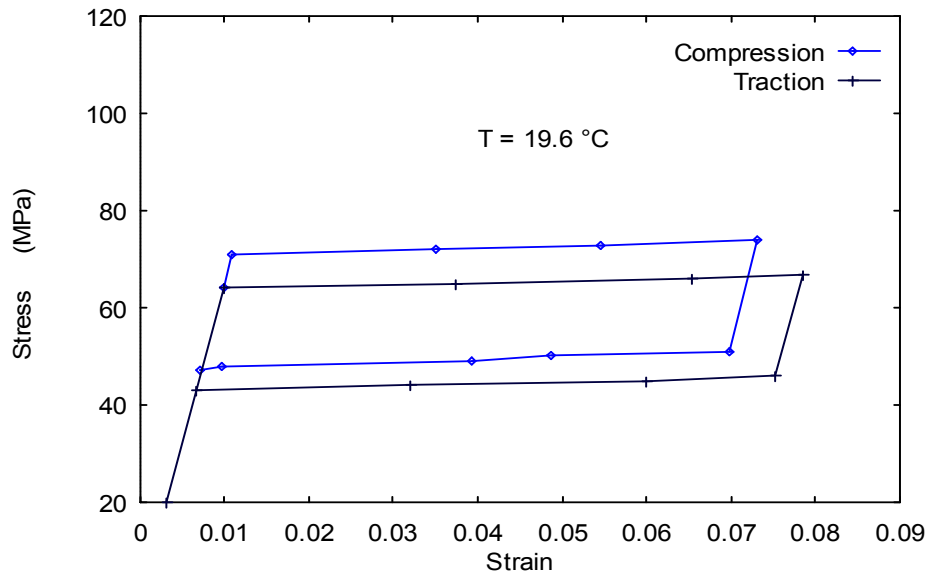


Figure 3: Pseudoelastic behaviour of the single crystal in uniaxial tension and compression.

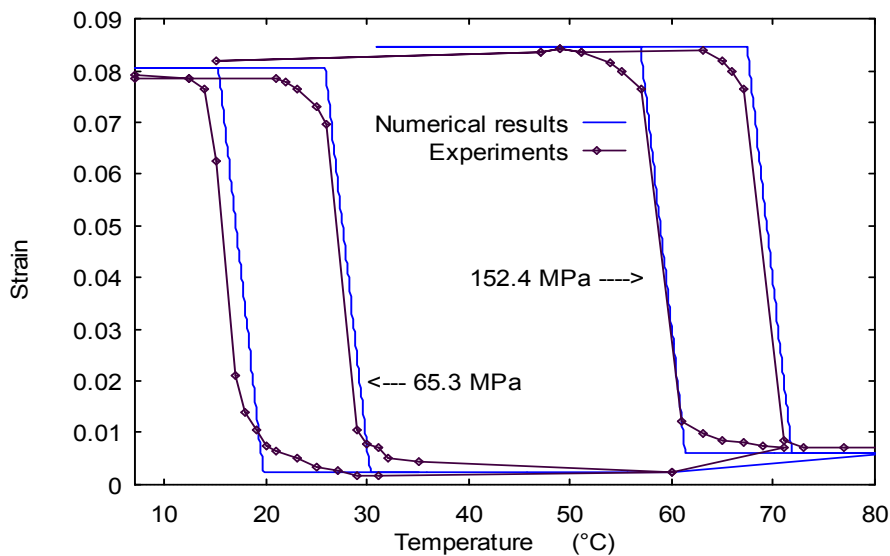


Figure 4: Hysteretic behaviour of the single crystal during heating-cooling at constant imposed stresses.

From the relation (37), we obtain the following relation joining Σ and T :

$$\frac{d\Sigma_{ij}}{dT} R_{ij}^h = \frac{B}{g} \tag{41}$$

We find again the law of Patel-Cohen [31] making the intrinsic parameters of the martensitic transformation and the mode of the applied thermomechanical loading intervene. The figure 5 represents the evolution of the martensite start and finish temperatures of transformation (M_s and M_f) in function of the imposed stress. This relation allows foreseeing the critical temperatures of transformation at a mechanical loading given in a test of cooling.

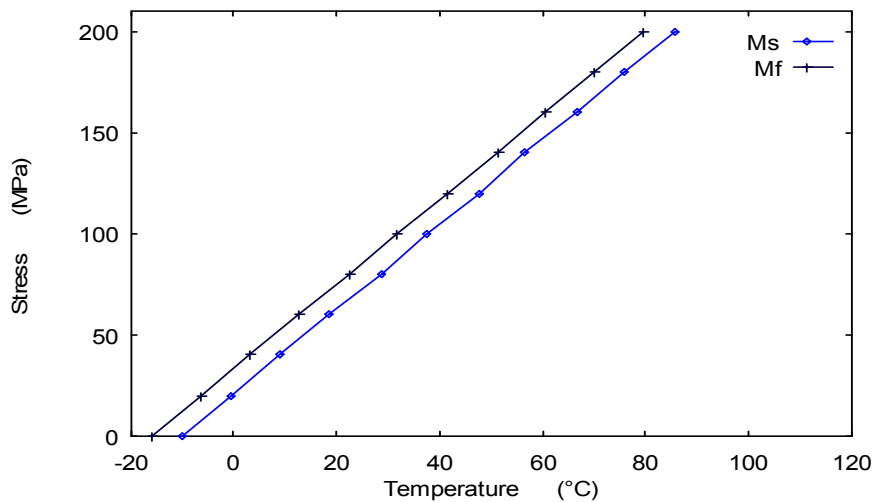


Figure 5: Evolution of the critical temperatures of the transformation (M_s and M_f) in function of the imposed stress during cooling.

7. CONCLUSION

The modelling developed in this work shows the importance of choice of the internal variables, which describe the material microstructural state and the role played by the internal stresses of incompatibilities in the SMA Thermomechanical behaviour. The modelling proposed for the dissipation energy already gives some useful indications on the thermomechanical response of the materials in martensitic transformation. In this modelling, we didn't study the interactions between the martensite variants, it will be interesting to study their effects on the global behaviour of single crystal in further research.

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