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Wavelet phase evaluation extended to digital speckle shearing interferometry

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Résumé
Cet article présente une application de la transformée en ondelettes continue en intérferomètrie de speckle à cisaillement ou shearographie. La méthode conduit directement à la dérivée seconde par rapport à une direction donnée de la phase à partir d'un shearogramme simulé représentant la phase (déformation).La dérivée première est obtenue par intégration. L'avantage de cette technique est de conduire à la dérivée seconde et à la dérivée première de la phase à partir d'une seule image tout en évitant le délicat problème du déroulage.

Abstract
This paper presents an application of the continuous wavelet transform in digital speckle shearing interferometry (DSSI) or Shearography. The method leads directly to the second derivative of the phase from simulated speckle shearing correlation fringes (shearograms). The phase gradient is obtained by integration. The advantage of the technique is to provide second derivative and first derivative of the phase from a single image without unwrapping step.

1. INTRODUCTION

Speckle interferometry and speckle-shearing interferometry offer a real time whole field method for analysing deformations and their derivatives. Applications of shearography include strain

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measurement, material characterisation, residual stress evaluation and vibration studies. Theory and a review of setups and applications are given in [1-4]. The second derivative and the first derivative of plane displacement are important in stress analysis. The aim of this paper is to apply the continuous wavelet analysis to extract simultaneously the second derivative and the phase gradient from simulated speckle shearing correlation fringes. The paper first describes in section 2 the method used to simulate the speckle effect. In section 3, we present the mathematical description of the DSSI technique. In section 4, we expose the wavelet phase evaluation method for speckle correlation interferometry. Finally, results of numerical simulations are presented in section 5.

2. SPECKLE EFEECT

In order to generate the speckle patterns diffracted by a surface, we simulate the clean imaging optical system [5] shown in the Fig.1 to generate the correlation fringes.

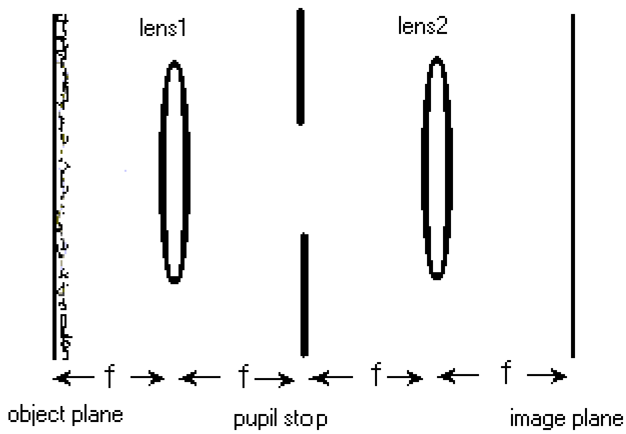


Figure. 1: Schematic setup

Within the paraxial approximation, optical propagation through any complex optical system, described by an ABCD ray transfer matrix, can be formulated by Collins formulas [6]. Collins has obtained an analytic form for the resulting complex field amplitude. Let $U(x_0,y_0)$ be the field on the input plane (x_0,y_0) located at z=0. The diffraction integral relating the fields across the input and output planes can be expressed as

$$U(x,y) = \iint U(x_0,y_0)H(x_0,y_0;x,y)dx_0dy_0 \tag{1}$$

Where (x,y) are transverse coordinates of the output plane and H is the propagation kernel given by:

$$H(x_0,y_0;x,y) = \frac{j}{\lambda B}\exp(-jkL)\exp[-\frac{jk}{2B}(D(x^2+y^2)-2(xx_0+yy_0)+A(x_0^2+y_0^2))] \tag{2}$$

where k is the optical wave number, j is a complex defined by $j^2 = -1$, λ is the free-space wavelength, L is the optical distance along the z axis, and A, B, D [7] are the ray matrix elements for the complete optical system between input and output planes. When the input and output planes are in free space, the determinant of ABCD matrix is unity (i.e. AD-BC=1).
To find the field in the image plane we first find the field at the plane of the limiting aperture, multiplying it by the corresponding aperture function, and applying the Collins formulation a second time to the remainder of the system. By varying the limiting aperture radius, we can match the target resolution to ensure a fully resolved speckle pattern across the observation plane.

3. DSSI TECHNIQUE

The basic principle of speckle interferometry is that the speckle pattern intensity distribution is a function of the relative phases of two interfering waves inside each resolution cell of an imaging setup. Displacement of the surface affects the intensity received in each speckle cell on the image. In shearography we combine the optical field scattered by the object with the same field shifted in a chosen optical shearing direction.
Let Δx be the shift, or shear, between the two interfering wavefronts in the x direction. Before deformation, the intensity distribution is given by:

$$I_1(x,y) = I_0(x,y)\left[1 + V(x,y)\cos(\Phi_s(x,y))\right] \tag{3}$$

where $\Phi_s(x,y)$ is the random phase between the two speckle fields

$$\Phi_s(x,y) = \phi_s(x,y) - \phi_s(x+\Delta x, y) \tag{4}$$

$\phi_s(x,y)$ and $\phi_s(x+\Delta x, y)$ being the random phases of the field scattered by the object and the shifted version of it respectively.
After a deformation, the intensity distribution becomes:

$$I_2(x,y) = I_0(x,y)\left[1 + V(x,y)\cos(\Phi_s^{'}(x,y))\right] \tag{5}$$

where

$$\Phi_s^{'}(x,y) = [\phi_s(x,y) + \varphi(x,y)] - [\phi_s(x+\Delta x,y) + \varphi(x+\Delta x,y)] = \Phi_s(x,y) - \frac{\partial\varphi(x,y)}{\partial x}\Delta x \tag{6}$$

The intensity distribution in the speckle shearogram can be expressed as:

$$I(x,y) = \left|I_1 - I_2\right| = 2I_0 V\sin\left(\frac{\partial\varphi(x,y)}{2\partial x}\Delta x\right)\sin\left(\Phi_s - \frac{\partial\varphi(x,y)}{2\partial x}\Delta x\right) \tag{7}$$

where $\varphi(x,y)$ is the additional phase change introduced by the deformation of the object I_0 is the bias intensity and V the visibility. Points with equal deformation difference (in shearing direction) are joined together by the interference fringes which is different from the classical methods in which points with equal deformation/amplitude are joined together. Basically, the shearogram represents the deformation gradient. We assume that the displacements are sufficiently small, that speckle decorrelation effects can be ignored. The grainy nature of speckle is conserved and the resulting correlation fringes always has a very noisy structure. The desired information containing $\sin(\frac{\partial\varphi(x,y)}{2\partial x}\Delta x)$ fringe term (shape of envelope modulating the random speckle term) may be rectified and filtered by appropriate computer image processing in order to remove the high frequency $\sin(\Phi_s - \frac{\partial\varphi(x,y)}{2\partial x}\Delta x)$ noise.

4. WAVELET PHASE EVALUATION TECHNIQUE

In this study, the phase mapping of speckle correlation fringes is obtained by our wavelet phase evaluation method [8]. From a modulated fringe pattern, the continuous wavelet transform is used to extract the localized spatial frequencies. The second derivative is performed simply from the modulus of the wavelet transform by extracting the extremum scales.

The one-dimensional wavelet transform of the modulated correlation fringe pattern intensity, in the x direction, is given by

$$W(y,s,\xi) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} I_0 V \psi^*_{s,\xi}(x)\,dy - \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} I_0 V \cos(mx + \Delta x\ \varphi'(x,y))\psi^*_{s,\xi}(x)\,dx \tag{8}$$

where $\varphi'(x,y) = \frac{\partial\varphi}{\partial x}(x,y)$, mx is the phase modulated carrier and $\psi^*_{s,\xi}(x)$ is the conjugate analyzing wavelet obtained for the shift ξ and the scale s.

Exploiting the wavelet localization property and assuming a slow variation of the intensity bias and the visibility, the wavelet transform becomes

$$W(y,s,\xi) = \frac{I_0(\xi,y)V(\xi,y)}{\sqrt{s}}\int_{-\infty}^{+\infty}\left[\cos(mx + \Delta x\,\varphi'(\xi,y) + (x-\xi)\Delta x\ \varphi''(\xi,y))\ \psi^*_{s,\xi}(x)\right]dx \tag{9}$$

By introducing Paul mother wavelet of order n formulated by

$$\Psi(x) = \frac{2^n\ n!\,(1 - i\,x)^{-(n+1)}}{2\pi\ \sqrt{(2\,n)!/2}} \tag{10}$$

and using Parseval identity, the modulus of the wavelet transform is equal to

$$\left|W(y,s,\xi)\right| = \left(\frac{I_0(\xi,y)V(\xi,y)m_1^n}{(2n)!}\right)s^{n+1/2}\exp(-sm_1) \tag{11}$$

where m_1 is the localized spatial frequencies given by

$$m_1 = m + \Delta x\varphi''(\xi,y) \tag{12}$$

The second derivative is performed simply from the modulus of the wavelet transform by extracting the extremum scales S

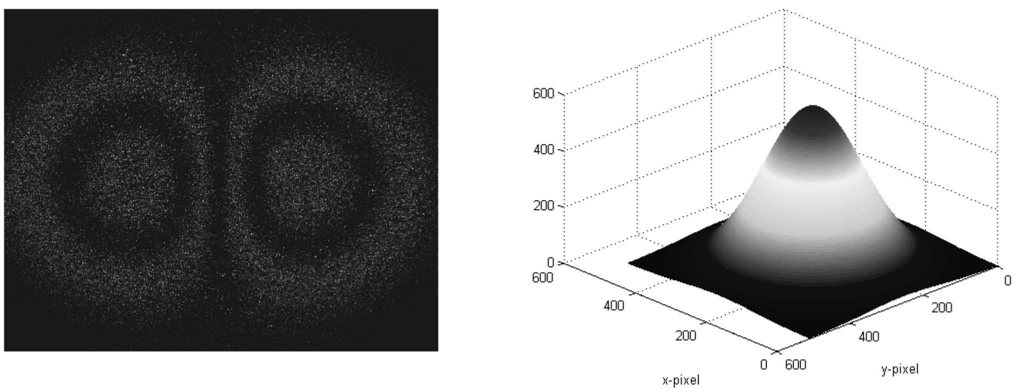
$$\Delta x\varphi''(\xi,y) = \frac{2\ n+1}{2\ S(\xi,y)} - m \tag{13}$$

$$\Delta x\frac{\partial^2\varphi}{\partial x^2}(\xi,y) = \frac{2\ n+1}{2\ S(\xi,y)} - m \tag{14}$$

This leads to the phase gradient by integration. To perform the wavelet transform we extend the fringe pattern at its left and right-band edges, using zero padding method. This step is necessary to avoid discontinuities and hence spurious, large values of the CWT at the edges of the data.

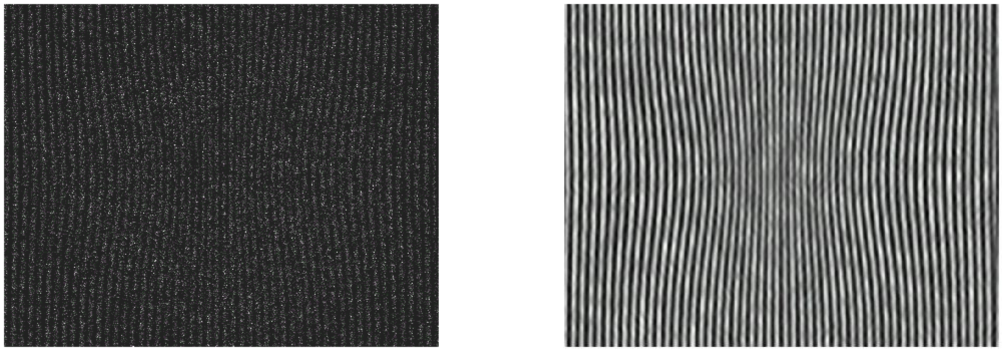
5. NUMERICAL SIMULATION RESULTS

The simulation consists in generating numerically speckle shearing correlation fringes of a given phase change resulting from surface displacement. We illustrated in Fig.2(a) the simulated speckle shearing correlation fringes obtained for the synthetic phase change presented in Fig.2.(b), with a shear Δx of 3 pixels for a speckle size of 2 pixels.



(a) (b)

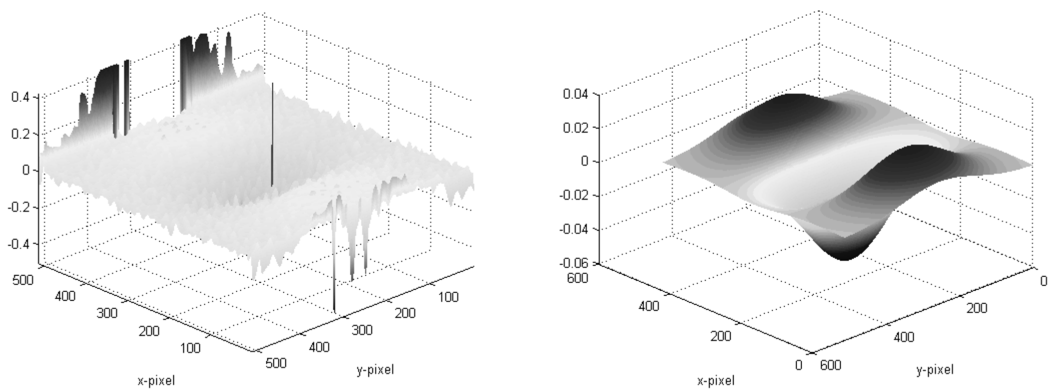
Figure 2: (a) The simulated speckle shearing correlation fringes, with a shear Δx of 3 pixels and a speckle size of 2 pixels. (b) The simulated phase change.



(a) (b)

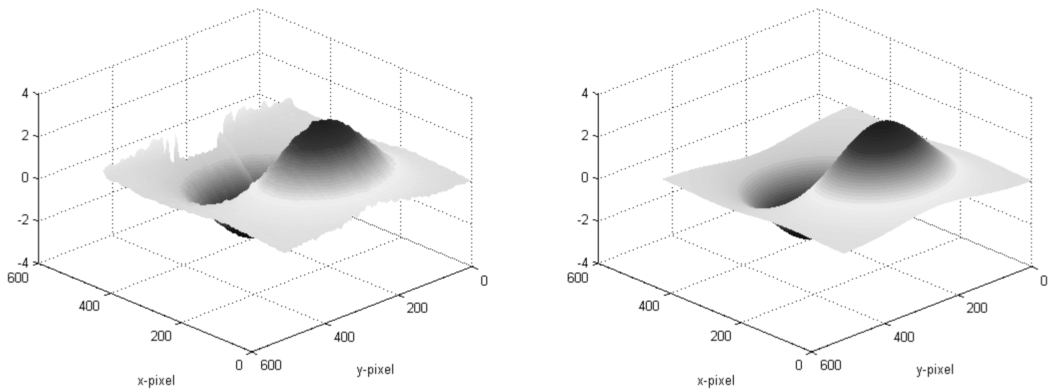
Figure. 3: (a) Noisily modulated speckle correlation fringes (b) Filtered Modulated speckle correlation fringes.

To perform the wavelet phase algorithm, the speckle noise must be removed from modulated correlation fringes by an adequate filtering process. In this study a discrete stationary wavelet transform SWT[9-11] is used for de-noising. The SWT performs a multilevel 2-D stationary wavelet decomposition. The noisily modulated shearing correlation fringes and his filtered image are shown in Fig.3.(a) and Fig.3.(b), respectively. Finally, simulation results are illustrated in Fig.4 and Fig.5 where the retrieved second derivative distribution is shown in Fig.4 while the obtained phase gradient distribution is presented in Fig.5.



(a) (b)

Figure.4 : (a) Recovered second derivative of the phase (b) Theoretical one



(a) (b)

Figure.5 : (a) Recovered phase gradient (b) Theoretical one

To evaluate the performance of the method, we have calculated the fidelity f defined by:

$$f = 1 - \frac{\sum_{1}^{N} (X - X_{ideal})^2}{X_{ideal}^2} \tag{16}$$

where X_{ideal} is the theoretical phase and X the recovered one.

A simple comparison of the exact proposed phase gradient and its derivative with the obtained results confirms the ability of this technique to extract, with a high accuracy, the second derivative and the phase gradient distribution from noisily correlation fringes. We have obtained a good result with a fidelity value f of 0.90 for the second derivative and a value of 0.99 for the phase gradient.
The simulation study and the SWT de-noising are computed by MATLAB.

6. CONCLUSION

In this study a simple method in order to simulate the speckle effect was presented and the wavelet phase evaluation method was extended to DSSI. The ability of this technique to extract, with a good accuracy, the second derivative was demonstrated. The phase gradient distribution is obtained simply by integration from a single shearogram , there is no need to perform the unwrapping process. We established that the stationary wavelet transform SWT used for de-noising is appropriate to remove the speckle noise.

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