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Finite volume method applied to the non-destructive testing of materials

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Résumé
Le contrôle thermique non destructif est capable de révéler la présence d'un défaut par la déformation qu'il produit sur la distribution du profil de température de surface et du flux à travers la paroi. La paroi est considérée, simple dans sa forme, comme une plaque plane dans laquelle est intercalée une couche de nature physique différente représentant un défaut. La plaque est soumise à une densité de flux uniforme à l'entrée. L'existence d'une anomalie change les propriétés thermiques de la paroi contenant le défaut, ainsi résulte une différence de température de surface entre la paroi à défaut et celle homogène. Les différences de température et de densité de flux sont analysées. Les courbes présentées incluent diverses situations pratiques et permettent d'optimiser la méthode d'inspection du matériau considéré. La sensibilité de la méthode de contrôle thermique non-destructif pour divers paramètres du système est observée.

Abstract
Thermal non-destructive control is able to reveal the presence of a defect by the deformation, which it produces on the distribution of temperature profiles of surface and flow through the wall. The wall is considered, simple in its form, as a plane plate in which is inserted a layer of different physical nature representing a defect. The plate is subjected to a uniform flow density at the entry. The existence of an anomaly changes the thermal properties of the wall containing the defect, thus results a difference in surface temperature between the wall and the homogeneous one. The differences of temperature and flow density are analysed. The curves presented include various practice situations and make it possible to optimise the method of inspection of material considered. The sensitivity of the method of thermal non-destructive control for various parameters of the system is observed.

1. INTRODUCTION

Thermal non-destructive control is a technique to obtain the surface temperature profile of a structure, then to establish a relationship between this information and the imperfections contained in this structure. If there is no internal source in the structure, the latter can be heated by an external one. The presence of the defect generally changes thermal flow through the structure due to the difference of the

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thermophysical properties of material and those of the defect. If the change of the flow in the sample is sufficient, the variation of temperature profiles of surface will be significant and can be detected.

The consultation of the literature relating to thermal non-destructive control indicates the very widespread applications of this technique of the detection of the vacuum in a metal, bending in an adhesive joint, the alveolar structures and the vacuum in ceramics, with the detection of the delamination and the cracks in the composites [2-5].

With an aim of understanding the technique of thermal non-destructive control and to be able to apply it quantitatively, its relationship with the thermal problem must be analysed. With the exception of certain problems of simple geometry, the analytical solution of the problem is extremely difficult to obtain. The digital techniques like the finite difference method and the finite element method are generally necessary for the majority of practical problems. The structure considered in the work is a plane plate containing a defect of inclusion as shows it figure 1. The defect can be a material whose properties are different from the structure containing it. If the defect is the air, this type of defect is called delamination or vacuum. In the preceding papers [4-7] we have introduced and showed the interest of the thermal impedance concept to the thermal non-destructive testing of the plane materials of the civil genius, where the study was centred on the defect position. In this work the problem is to evaluate the surface temperature of the structure with defect and to compare it with the homogeneous one, when the plate is subjected to a uniform flow density at the entry.

2. FORMULATION OF THE CONDUCTION PROBLEM

2.1 The model

Let us consider the case, of a plane wall of thickness l=20 mm, height Yl=10 mm, and initial temperature T_0=298k, in which is inserted a layer of different nature representing a defect. The discontinuity has a thickness ($e_d=l_2-l_1$), a height Yl=10 mm, placed at the distance l_1 of the input face x=0 of the wall. The preceding geometrical and thermophysical parameters are arbitrary taken. Except the defect, the material of the wall is homogeneous and the thermal properties are constant but different from those of the defect. This configuration is schematised by figure 1.

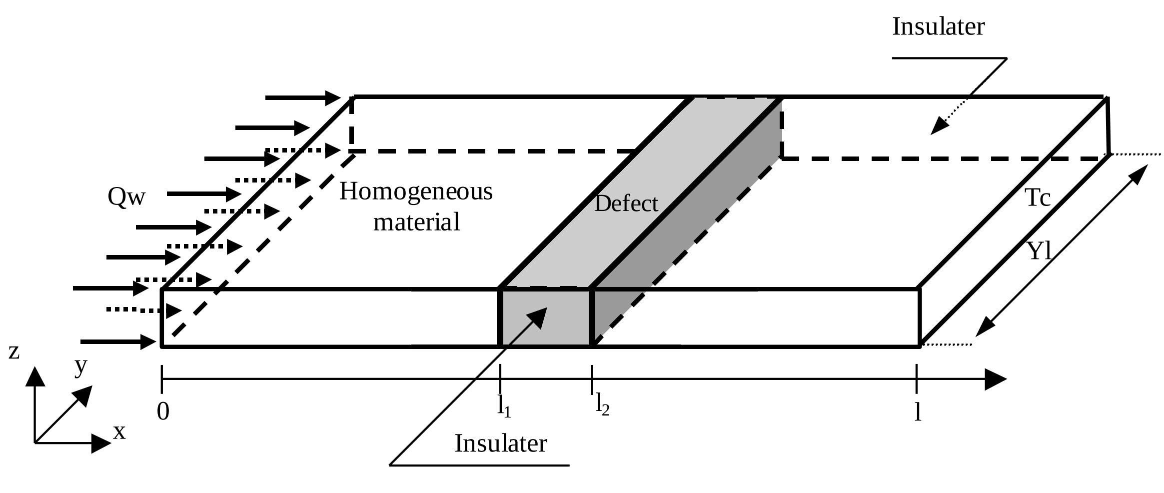

Figure 1: The studied model

The wall is definite between plans x=0 (plane from input where an external heat source applies a uniform thermal flow density Qw=1000w/m^2 at t=0), and x=l (output plane, where the temperature is maintained constant), surfaces y=0 and y = Yl are isolated. For this model we suppose that the thermal resistance of contact between material and the defect is negligible. The defect is materialised by a polystyrene (k=0,041 w/m.k (thermal conductivity), ρ=14,7 kg/m^3 (density) and c=1310 J/kg.k (specific heat)) plate inserted in a concrete (k=1,7 w/m.k, ρ=2250 kg/m^3 and c=827 J/kg.k) wall.

For each type of defect, insulator or conductor, we considered a certain number of configurations related to the following parameters: position, nature and dimensions of the defect. The choice of these configurations is completely arbitrary but describes the majority of the possible cases relating to these parameters. The temperature at the entry, of homogeneous material (without defect) is defined by $T_{1S}(t)$ and that with defect by $T_{1A}(t)$, also the flow density at the output of material without defect by $Q_{2S}(t)$ and that with defect by $Q_{2A}(t)$. The presence of the defect brings a difference in temperature:

$$\Delta T(t) = T_{1A}(t) - T_{1S}(t) \qquad \text{with} \qquad \Delta T(0) = 0 \tag{1}$$

and of flow density at the output:

$$\Delta Q(t) = Q_{2A}(t) - Q_{2S}(t) \qquad \text{with} \qquad \Delta Q(0) = 0 \tag{2}$$

This difference in temperature is due to the presence of the defect and implies different values of thermal flow in the structure. Its detection, like that of the flow density, is the major concern of non-destructive testing.

2.2 General equation of heat conduction

The equation of the thermal diffusion in two-dimensional, homogeneous medium, and without internal source is [1]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) \tag{3}$$

where the quantities ρ, c and k stand for the density, specific heat and conductivity of the material respectively.

The initial and boundary conditions are:

$$
\begin{array}{llllll}
-k \dfrac{\partial T}{\partial x} = Q_w = 1000 w/m^2 & \text{at} & x = 0 \ \forall y & & \\
-k \dfrac{\partial T}{\partial y} = 0 & \text{at} & y = 0 & \text{and} & y = Y1 \ \forall x \\
T(x, y, t) = T_c = 298 k & \text{at} & x = 1 \ \forall y & \text{and} & t \\
T(x, y, t) = T_0 = 298 k & \text{at} & t = 0 \ \forall x & \text{and} & y
\end{array}
$$

in all the domain defined by the figure 1.

2.3 Numerical Method of resolution

For the resolution of the equation of heat, we used the method suggested by PATANKAR [1]. The method consists in cutting out the integration domain (material with defect) in a group of small elementary volumes or "control volumes" (figure 2), surrounding the point of grid, on which we integrates the differential equations.

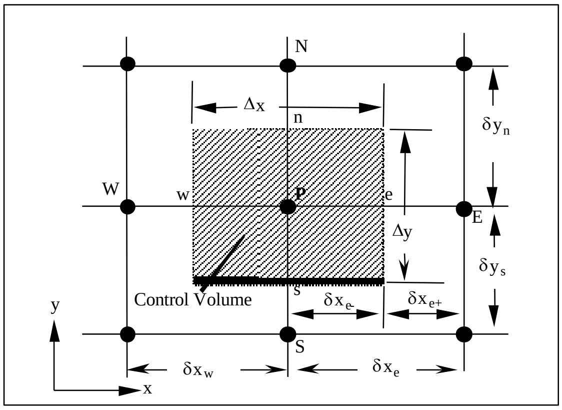

Figure 2: Two dimensional control volume

2.4 Discretization in transient state

The integration of the heat equation on the volume of control shown in figure 2 and in the interval of time ranging between t and t+Δt gives the following discretized equation [1] for a node P:

$$a_p T_p = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b \tag{4}$$

$$a_E = \frac{k_e \Delta y}{(\delta x)_e} \qquad a_w = \frac{k_w \Delta y}{(\delta x)_w} \qquad a_s = \frac{k_s \Delta x}{(\delta y)_s}$$

$$a_N = \frac{k_n \Delta x}{(\delta y)_n} \qquad a^\circ_p = \rho c \frac{\Delta x \Delta y}{\Delta t} \qquad b = a_p^0 T_p^0$$

$$a_p = a_E + a_W + a_N + a_S + a_p^0$$

where T_p^0 is the old value (at time) of T_p. The discretized boundary conditions are:

- Input

$$a_p T_p = a_E T_E + b$$

where: $a_P = a_E = \dfrac{k_e \Delta y}{(\delta x)_e}$, $b = Q_w \Delta y$.

- Output

$$a_p T_p = a_W T_W + b$$

where : $a_P = 1$, $a_W = 0$, $b = T_P = T_c$.

- Insulaters faces

$$a_p T_p = a_N T_N + b$$

where: $a_P = a_N = \dfrac{k_n \Delta x}{(\delta y)_n}$, $b = 0$

$$a_p T_p = a_S T_S + b$$

where: $a_P = a_S = \dfrac{k_s \Delta x}{(\delta y)_s}$, $b = 0$

The system of equations that one can write at the structure level, can get in the general case under the following matrix shape:

$$[A]\{T\} = \{Q\}$$

where $[A]$ is the tridiagonal matrix of exchange coupling at the level of the wall, $\{T\}$ is the unknown temperature vector and $\{Q\}$ is the vector of coupling.

The mondimensional equations are solved directly by TDMA algorithm (TriDiagonal-Matrix Algorithm) [1]. We cannot extend TDMA algorithm to the resolution of the two-dimensional algebraic equation. From where the choice of an iterative method of resolution named "method of line by line" [1] in which TDMA algorithm occupies the greatest part.

A previous survey of the stability drove us to adopt following space and time steps: Δx=0,25mm, Δy=0,25mm, and Δt=0,6s. Calculations are made with a precision inferior to 10^{-6}.

3. RESULTS AND DISCUSSION

The difference in temperature $\Delta T(t)$ gives an estimate on the required resolution of the equipment of thermal non-destructive control. If the amplitude of ΔT is low, detection is relatively difficult and requires a very sensitive detector.

The curves of temperatures, show that the differences of temperatures $\Delta T(t)$ and the flow densities $\Delta Q(t)$ pass by an extremum, (figure 3a, 3b), during the interval of control.

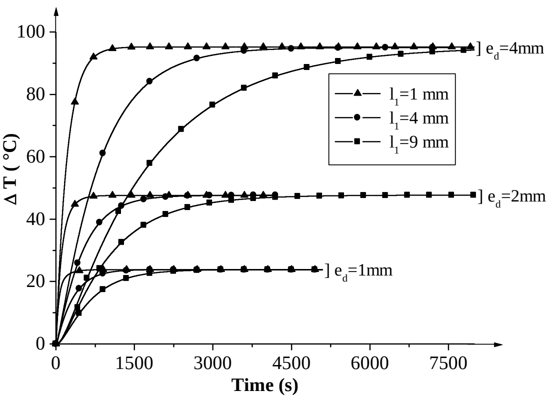

Figure 3a: influence of position and thickens of defect on the temperature difference $\Delta T=T_{1A}-T_{1S}$.

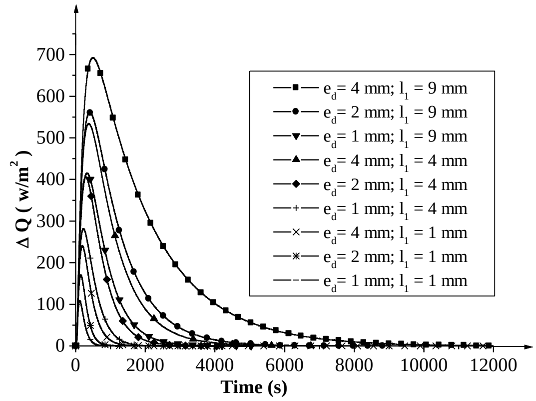

Figure 3b: influence of position and thickens of defect on the flow density difference $\Delta Q=Q_{2A}-Q_{2S}$.

This extremum represents optimal time to make control, it is the moment to which appears the maximum difference in temperature meaning the presence of defect. With the greatest values of time, the temperature T_{1S} and T_{1A} tighten both towards the steady state and consequently $\Delta T(t)$ tends towards a constant value (figure 3a).

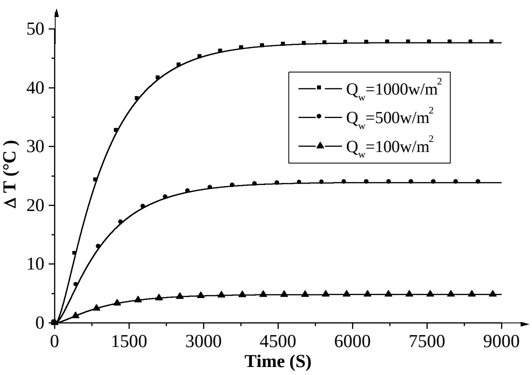

Figure 4a: influence of the external source of energy on the temperature difference $\Delta T=T_{1A}-T_{1S}$ with e_d=2mm and l_1=9mm.

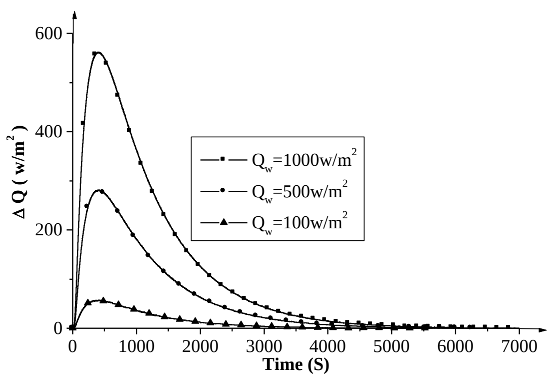

Figure 4b: influence of the external source of energy on the flow density difference $\Delta Q=Q_{2A}-Q_{2S}$ with e_d=2mm and l_1=9mm.

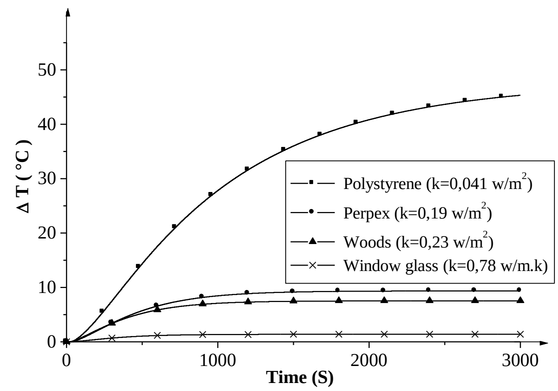

Figure 5a: influence of the conductivity on the temperature difference $\Delta T=T_{1A}-T_{1S}$ with e_d=2mm and l_1=9mm.

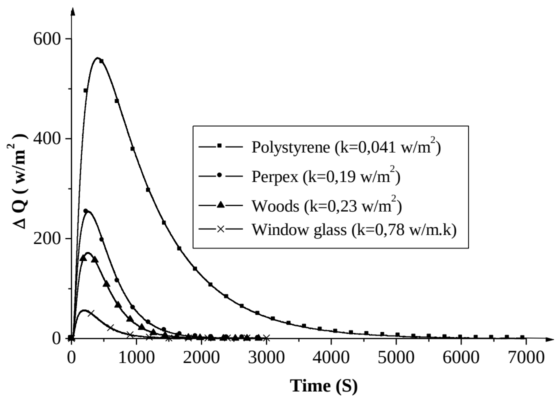

Figure 5b: influence of the conductivity on the flow density difference $\Delta Q=Q_{2A}-Q_{2S}$ with e_d=2mm and l_1=9mm.

A great value of ΔT(t) or of ΔQ(t) can results from a great value of the flow density Qw as shows it the figures 4a and 4b or of a great difference between conductivity of homogeneous material and the defect as shown in the figures 5a and 5b, ΔT will be positive if k defect great than k material and negative in the opposite case. On the other hand the detection of the defect is relatively difficult if the heat flow applied to the face input is very weak. Therefore, a great value of ΔT permits an easy detection of defect. We can see starting from the figure 3a that in transient state ΔT(t) depends of depth l_1 and the thickness ($e_d=l_2-l_1$). Towards the steady state the depth of the defect has however a negligible effect on ΔT(t) for a constant thickness of the defect. Thus the thickness of the defect can be estimated starting from the steady state by reversing the face of excitation (opposite face) of material and the depth corresponding of the defect in the material from the transient state.

4. CONCLUSION

When we apply a thermal excitation to the plate of the studied model, the value of thermal flow changed through this one because of the presence of the internal defect. It results a deformation of profile surface temperatures, which can be detected and associated to the defect. The model, which can be used for various structures, is compound of a plate with a defect of inclusion. The elaborated data-processing program can treat several possible configurations. The difference ΔT(t) always has a maximum or a minimum during the test according to the defect nature. The maximum corresponds to a resistive defect and the minimum to the capacitive one. The maximum represents the informative time of the presence and the position of the defect. Also, more the difference between conductivity of the structure and the defect is great, more the value of ΔT(t) is large. A significant thermal source is necessary. The thickness of defect can be evaluated from permanent regime, and the position can be determined in transient regime. The analysis can be used in the establishment of the equipment parameters of thermal non-destructive control.

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