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Unsteady natural convection with viscosity variation in two-dimensional square

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Résumé

L'objectif de cette étude consiste à déterminer l'effet de la variation de la viscosité en fonction de la température sur les écoulements de la convection naturelle instationnaire des fluides Newtoniens à propriétés physiques variables dans une cavité différentiellement chauffée. L'analyse du présent travail est basée sur une résolution numérique des équations différentielles décrivant le présent problème utilisant une technique de différence finie de procédure implicite aux directions alternées (*ADI*). L'étude du régime transitoire a montré que les champs de vitesse et de température atteignent tous une valeur maximale puis décroissent vers une valeur stationnaire. Ce phénomène de dépassement subit des amplifications ou des atténuations selon les valeurs attribuées aux paramètres caractérisant la variation de la viscosité.

Mots-clés: Convection naturelle instationnaire, Variation de la viscosité, Milieu confiné, Différence finie

Abstract

Steady and transient laminar two-dimensional natural convection of a Newtonian fluid with variable properties in a square cavity is investigated. The temperature-dependent viscosity effect was studied. The results are obtained by solving numerically the partial differential equations describing the conservation of mass, momentum, and energy of the transient natural convection in a cavity with differentially heated end walls using an alternate implicit finite difference *(ADI*) method. The characteristic transient velocity and temperature rise to a peak and decrease to a steady value. This overshoot phenomenon is amplified or attenuated depending on the values selected for the dimensionless parameters characterizing the viscosity variation.

Key-Words: Unsteady natural convection, Viscosity variation, Confined medium, Finite difference discretization

1. INTRODUCTION

The transport of heat or mass by buoyancy-induced convective motions is a mechanism relevant in many physical systems. During these last years, numerous theoretical, experimental and numerical studies have been constituted to understand various aspects of natural convection flows. Recently natural convection in enclosed cavities has been extensively documented and widely studied both numerically and experimentally. The study of natural convection in enclosures is essential in many industrial and geophysical problems. Some examples of natural convection are motion of fluid in

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computer equipment, heat exchangers and various types of radiators. These applications require knowledge of natural convection in various contexts. Yeoh and al [1] have recently studied the natural convection and interface shape in crystal growth by numerical and experimental methods. In particular, the idealized problem of steady laminar flow end walls has been extensively studied. Many papers have been published. Patterson and Imberger [2] have given a number of possible transient flow types, for nondimensional parameters describing the flow such as the Rayleigh number R_a , the Prandtl number P_r and the temperatures of the cooled and heated wall. The flow regimes can be conductive or convective depending on the relative values of R_a. Also, Gilly and al [3] have studied the effect of thermal wall conditions on the natural convection in a vertical rectangular differentially heated wall; two models of non-uniform temperature distribution have been studied by a numerical simulation. Steady and transient laminar two-dimensional natural convection of a fluid in an inclined rectangular enclosure were numerically studied by Talaie and Chen [4], the effect of inclination angle on the flow development was discussed.

Particular attention has been directed in the present study on the flow of temperature dependent-properties of a fluid in a square cavity. Sehyn and al [5] have investigated the influence of variable viscosity of temperature-dependent fluids on the laminar heat transfer in a rectangular duct. Karl and al [6] have also interested to the influence of the viscosity variation on the convection in a horizontal fluid layer with isothermal boundaries. Their results, based on exponential and upper-exponential viscosity variations, have showed that the viscosity variation has a significant effect on the flow field. Etmad and Mujundar [7] have employed a numerical scheme based on Galerkin's finite element method to solve the three-dimensional governing equations of the problem in order to bring out many interesting effects of variable viscosity and viscous dissipation on laminar convection heat transfer of a power law fluid in the entrance region of a semi-circular.

The present study is undertaken in order to gain an understanding of certain aspects of convective transport of a fluid with variable viscosity. Thus, the purpose of this investigation is to study numerically the influence of the viscosity variation on transient natural convection in a confined duct. This present analysis shows that the viscosity variation has a significant effect on the flow patterns, the transient velocity and temperature profiles rise to maximum values, next they decrease to their steady values. This overshoot phenomenon was observed and studied by Sammakia and Gebhart [8] for a vertical flat surface, Maslouhi and Robillard [9] for a vertical plate with the viscosity variation and also by Bellahmar and Maslouhi [10] in two-dimensional enclosure. This phenomenon is amplified or attenuated depending on the viscosity variation coefficient.

2. FORMULATION OF THE PROBLEM

A schematic diagram of the geometry of the problem is shown in figure 1. It is assumed that the third dimension of the enclosure is large enough so that the flow and heat transfer are two-dimensional. The bottom and the top boundaries of the enclosure are considered to be adiabatic, the left and the right boundaries are isothermal at constant temperature T_h and T_c respectively. It is assumed that the fluid inside the enclosure is Newtonian, the motion of the fluid is laminar and all the fluid properties are constant except for the viscosity and density, which are linearly dependent on the temperature. Further, the viscous dissipation of the thermal energy is negligible. The characteristic parameters of the problem are the Rayleigh number R_a based on temperature difference ($\Delta T=T_h-T_c$), the Prandtl number P_r and the viscosity variation coefficient ξ. The Boussinesq approximation is valid. In this approximation, the allowable temperature difference is quite small for fluids.

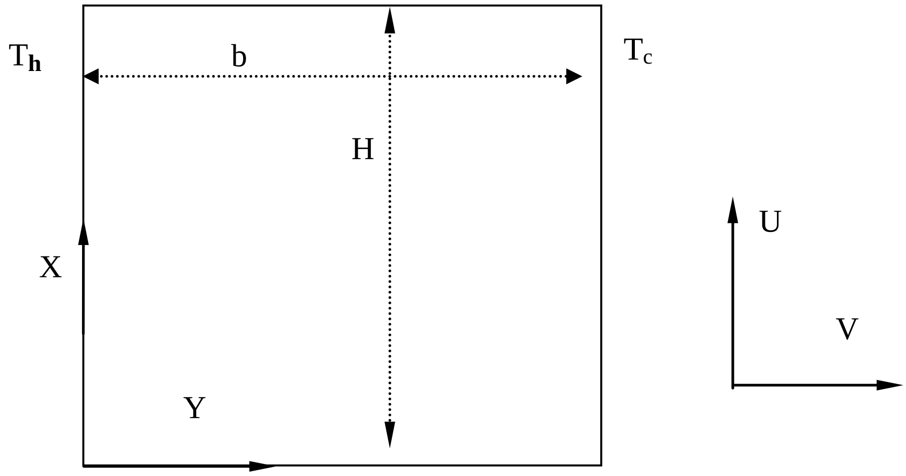


Figure 1: Scheme of the cavity

3. FLUID PROPERTIES

For liquids, all transport properties vary with temperature. However, for many liquids, for examples, glycerine, petrol oil, silicone oil …, except for the viscosity, the variation of properties is negligible. In fact, the assumption of only variable viscosity is realistic and applicable to many liquids. Many studies have considered the viscosity as varying linearly or exponential with the temperature. In this paper, we choose a linear viscosity-temperature variation used and described in detail in [11, 12], especially when the temperature differences are of the 20°C order. Piau [13] has already demonstrated that the viscosity of most liquid, with small temperature difference, can be approximated by a Taylor's expansion:

$$\mu = \mu_r \left[1 + \frac{1}{\mu_r} \left(\frac{d\mu}{dT} \right)_r (T - T_r) \right] \tag{1}$$

where μ_r and $\left(\frac{d\mu}{dT} \right)_r$ are respectively the viscosity and the viscosity variation coefficient at the referential temperature $T_r = T_c$.

As in the usual manner, we consider a linear temperature dependence of the fluid density:

$$\rho = \rho_r \left[1 - \beta (T - T_r) \right] \tag{2}$$

where the volumetric expansion coefficient β is given by :

$$\beta = -\left(\frac{1}{\rho_r} \right) \left(\frac{\partial \rho}{\partial T} \right)_r$$

and ρ_r is the fluid density at the referential temperature T_r

4. MATHEMATICAL MODEL

Under the Boussinesq approximation for computational purposes, transient natural convection is governed by partial differential equations expressing the conservation of mass, momentum and energy. In order to eliminate the pressure, we introduce the stream function ψ and the vorticity ω. Using vorticity stream function formulation, the governing equations can be written in the following dimensionless form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{3}$$

$$\frac{\partial \Omega}{\partial \tau} + U\frac{\partial \Omega}{\partial X} + V\frac{\partial \Omega}{\partial Y} = \left(P_r + \xi\theta\right)\left[\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2}\right] - \xi\left[\left(\frac{\partial^2 \theta}{\partial X^2}\right)\left(\frac{\partial V}{\partial X}\right) - \left(\frac{\partial^2 \theta}{\partial Y^2}\right)\left(\frac{\partial U}{\partial Y}\right) + 2\left(\frac{\partial^2 \theta}{\partial X \partial Y}\right)\left(\frac{\partial V}{\partial Y}\right)\right] + R_a P_r\left(\frac{\partial \theta}{\partial Y}\right) \tag{4}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \tag{5}$$

where

$$\Omega = -\nabla^2 \Psi \tag{6}$$

The stream function Ψ is defined by:

$$U = -\frac{\partial \Psi}{\partial Y} \qquad V = \frac{\partial \Psi}{\partial X} \tag{7}$$

The equations have been normalised using the following equations:

$$\tau = \frac{\alpha t}{b^2}; \quad X = \frac{x}{b}; \quad Y = \frac{y}{b}; \quad U = \frac{ub}{\alpha}; \quad V = \frac{vb}{\alpha}$$
$$\Psi = \frac{\psi}{\alpha}; \quad \Omega = \frac{\omega b^2}{\alpha}; \quad \theta = \frac{T - T_c}{T_h - T_c}; \quad \Delta T = T_h - T_c \tag{8}$$

where τ is the dimensionless time, U and V are the components of the velocity and α is the thermal diffusivity.

Three dimensionless parameters are introduced in vorticity-stream function formulation (4):

the Prandtl number, characterizing the nature of considered fluid

$$P_r = \frac{\mu_r C_p}{k}$$

the Rayleigh number, characterizing different types of the flow

$$R_a = \frac{g\beta b^3 \Delta T}{\alpha\mu}:$$

and the viscosity variation coefficient

$$\xi = \left(\frac{d\mu}{dT}\right)_r \left(\frac{\Delta T}{\rho\alpha}\right)$$

where C_p, k and g are respectively the specific heat at constant pressure, the thermal conductivity and the gravitational acceleration.

$\xi<0$: indicates that the viscosity of the fluid decreases near the heated wall.
$\xi>0$: indicates that the viscosity is amplified near the heated wall.

The initial and boundary conditions are:

$$\begin{array}{llll}
\tau = 0: & \Psi = U = V = \Omega = \theta = 0 & & \\
\tau > 0: & X = 0: & \Psi = U = V = \Omega = 0, & \frac{\partial\theta}{\partial X} = 0 \\
X = 1: & \Psi = U = V = \Omega = 0, & \frac{\partial\theta}{\partial X} = 0 & \\
Y = 0: & \Psi = U = V = \Omega = 0, & \theta = 1 & \\
Y = 1: & \Psi = U = V = \Omega = 0, & \theta = 0 &
\end{array} \quad (9)$$

Equations (4) and (5) are subjected to the conditions on ψ and its first derivations which vanish at the boundaries and on θ or its normal derivative which are specified at the boundaries. The vorticity boundary conditions can be obtained from the velocity boundary conditions as:

$$\Omega_j = -\frac{2\Psi_{j-1}}{(\Delta Y)^2}$$

The subscripts "j" and "j -1" denote respectively the values of the variables at the boundary and at the node next to the wall into the enclosure, ΔY is the grid size at the boundary.

5. NUMERICAL RESULTATS AND DISCUSSIONS

The coupled motion and energy, equations (4) and (5), are non-linear, second-order partial differential equations of parabolic type. The numerical methods as alterning implicit method and others can be used to solve these equations. In this study a two-dimensional alterning direction implicit (***ADI***) procedure is employed. Central differences formula are used for the spatial derivation terms, in the vorticity, energy and Poisson equations. An alternate direction implicit procedure is adapted to obtain from equations (4) and (5) the vorticity and temperature profiles. The finite difference forms of the vorticity and energy equations are written in conservative form so that the convective terms preserve the conservative property [14]. The elliptic equation (6) for the stream is solved by a successive over-relaxation procedure (***SOR***). The velocities at all grid points are determined with equation (7) using updated values of the stream function. The calculations is carried out using a 30x30 mesh. The results showed that the use of a fine mesh made relatively little difference to the final results. For each time step, the convergence criterion [15] imposed is:

$$\frac{\sum_{ij}\left|\Psi_{ij}^{n+1}-\Psi_{ij}^{n}\right|}{\sum_{ij}\left|\Psi_{ij}^{n+1}\right|}\leq 10^{-4}$$

Various time step sizes have been tested and the value 10^{-4} was selected. The accuracy of the numerical model is verified by comparing the results from the present investigation to several ones reported in [4]. The obtained numerical results, in the present study, have revealed that, depending on the Rayleigh number, different modes of convection may be observed inside the cavity. Numerical solutions are obtained for steady and transient natural convection in a square cavity.

5.1 Streamlines, isotherms and velocity distributions

Steady state

The preliminary results obtained for constant properties indicate that increasing R_a, the description of the flow changes from conduction dominated to convection dominated, these regimes were obtained by Mallinson and DeVahl Davis [16]. The isotherms and streamlines for $R_a = 10^4$, $P_r = 1$ and $\xi=0$ are shown in figures 2 and 3. The results of these figures are in good agreement with those of Talaie [4].

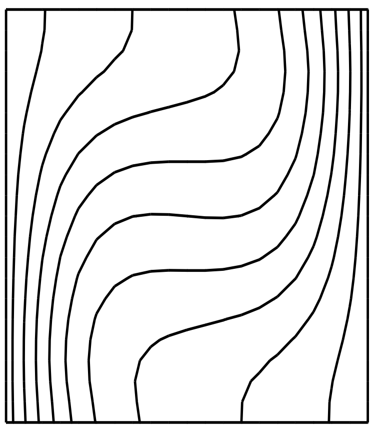


Figure 2: Streamlines for P_r=1, R_a=10^4 and ξ = 0 Comparison with Talaie [4]

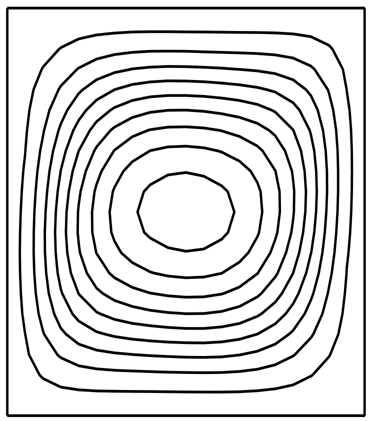
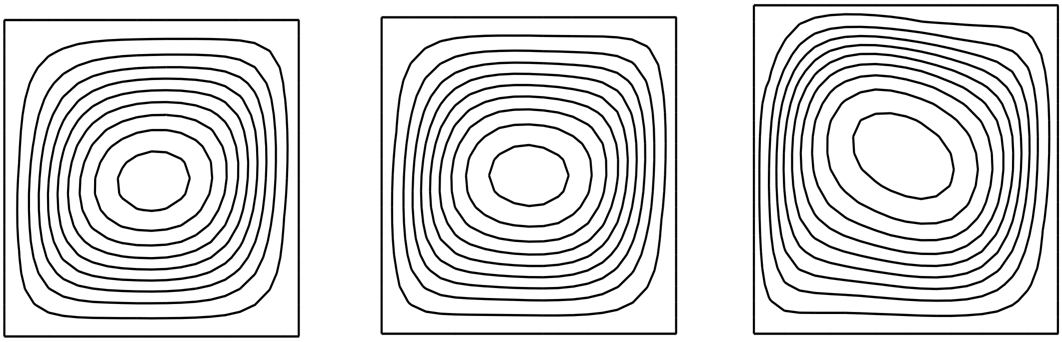


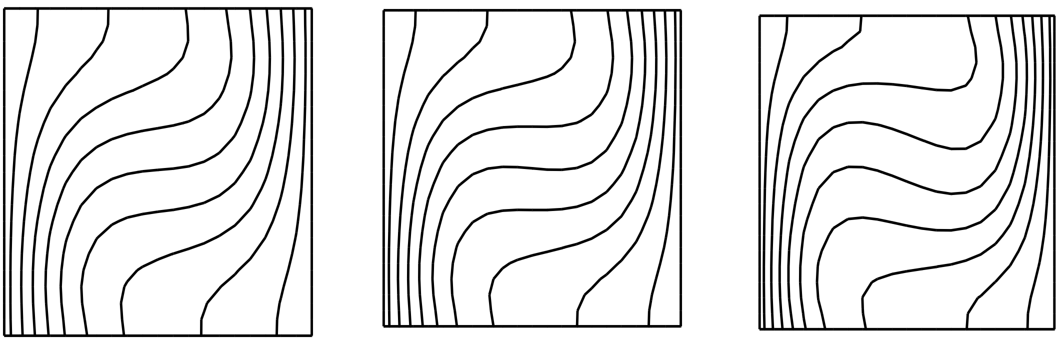
Figure 3: Isotherms for P_r=1, R_a=10^4 and ξ = 0 Comparison with Talaie [4]

Figure 4 gives the streamlines for natural convection of a fluid for P_r=1, R_a=10^4 and for ξ= ±1; 0. In the case of a linear variation of the viscosity with the temperature, we present numerical results for a range of their viscosity parameter which is compatible with the viscosity ratio range used by Carey and Mollendorf [11,12]. These results illustrate the effect of the viscosity variation on the flow structure, the convection motion depends of the relative value of ξ. For negative value of ξ, the effect of convection increases, it can be attributed to the decrease of the viscosity near the heated wall, which brings out the significant increase in velocity gradients and subsequent decrease in fluid temperatures near the wall. Figure 5 shows the influence of the viscosity variation on the isotherms, the results indicate that the tickness of the boundary layers at the vertical walls decreases for negative value of ξ (case ξ<0). It is observed that the isotherms are distorted because at negative value of ξ the convection in the enclosure becomes strong.



$\xi = 1$ $\xi = 0$ $\xi = -1$

Figure 4 : Streamlines for $P_r = 1$, $R_a = 10^4$ and for three relative values of ξ



$\xi = 1$ $\xi = 0$ $\xi = -1$

Figure 5 : Isotherms for $P_r = 1$, $R_a = 10^4$ and for three relative values of ξ

Figure 6 shows the effect of viscosity-dependent temperature on the dimensionless X-component of velocity U at the midplane of the enclosure, for a fluid with $P_r = 1$, $R_a = 10^4$ and $\xi = \pm 1; 0$. It can be seen that the flow is more strongly for the negative values of viscosity contrast when compared with case $\xi = 0$ and $\xi = 1$. The negative coefficient of viscosity variation $\xi = -1$ indicates that the viscosity of the fluid decreases near the heated wall, thus, the transfer of mass is amplified which causes higher velocity also higher velocity gradients near the wall, whereas for the case where ξ is positive, there are most viscosity near the heated wall, then, the fluid motion is reduced. It is observed that the amplification of the velocity is more pronounced at the heated wall. In addition, the results of figure 7 give the importance role of viscosity-dependent temperature on temperature field, the contribution of natural convection increases for the negative values of viscosity contrast.

Transient study

The transient analysis shows clearly the temperature and dynamic fields increase progressively to a peak and subsequently decrease to reach constant values, which correspond to the steady state of the flow.

Figure 8 gives the vertical velocity evaluated at the midplane of the enclosure near the heated wall for constant viscosity. This result shows that the steady state of these flows is independent of the value of the Prandtl number for constant properties and which generally agree with Patterson [2]. Initially, the fluid near the left begins to be heated, the transient conduction is a dominant heat transfer mode. Therefore, the velocity increases rapidly to a peak, and drops subsequently to the steady value, this fact is due to convection effect. This overshoot phenomenon is also found out in a vertical heated plate by Mahajan and Gebhart [17], Langham [18, 19], Maslouhi [20] for a vertical surface and in a confined duct by Bellahmar and Maslouhi [21,22].

For the case of variable viscosity, the results of figures 9 and 10 indicate the transient evolution of the velocity fields for $P_r = 1$, $R_a = 10^4$ and $\xi = \pm 1; 0$. It is found that the evolution illustrates different flow regimes:

-Initial transient regime corresponding to the pure conduction.
-Intermediate regime corresponding to the overshooting phenomenon.
-Steady regime.

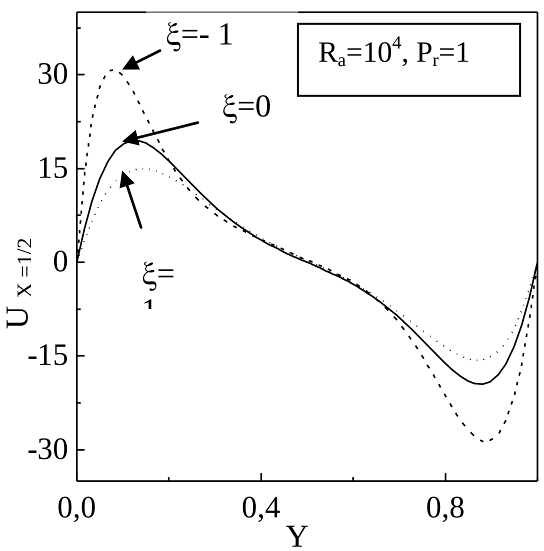


Figure 6: Velocity profile along Y for various ξ
$\xi = 1$; $\xi = 0$; $\xi = -1$

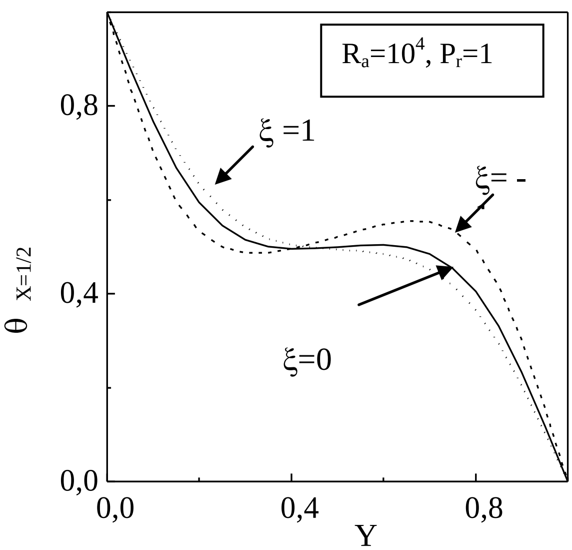


Figure 7: Temperature profile along Y for various ξ
$\xi = 1$; $\xi = 0$; $\xi = -1$

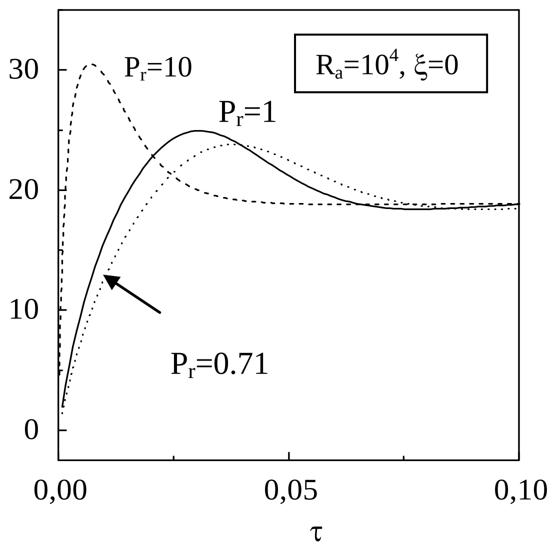


Figure 8 : Transient velocity at the midplane of the cavity near the heated wall

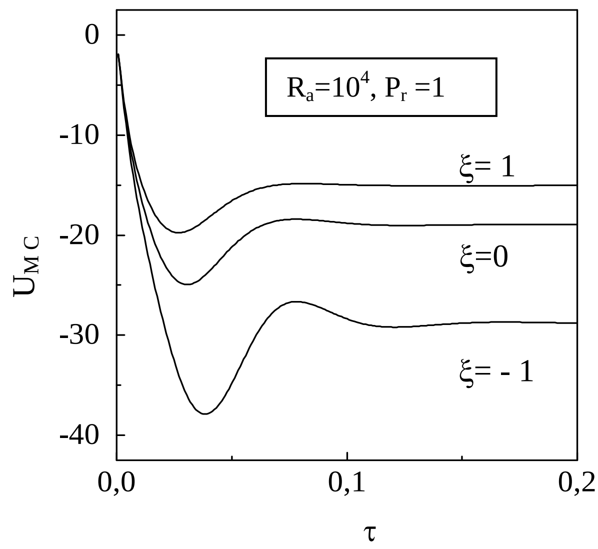


Figure 9: Transient velocity at the midplane of the cavity near the cooled wall for various ξ

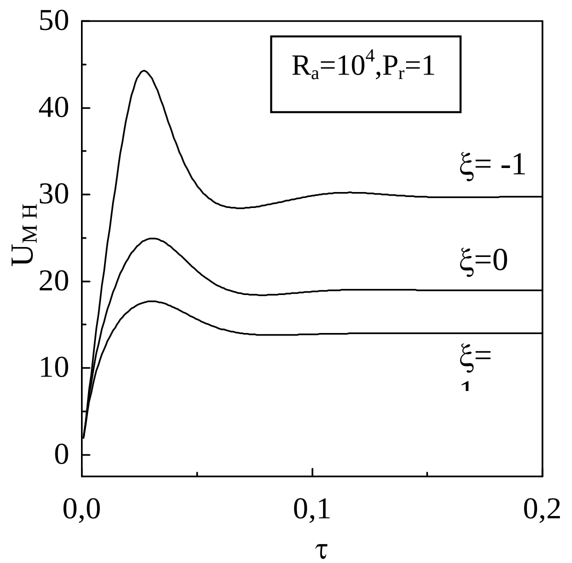


Figure 10: Transient velocity at the midplane of the cavity near the heated wall for various ξ

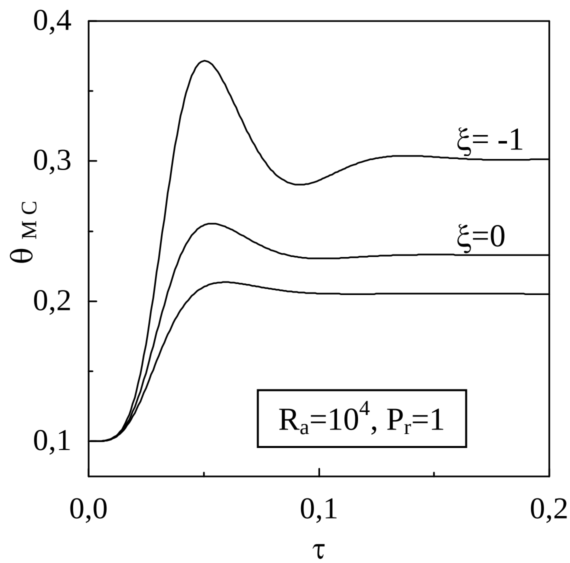


Figure 11: Transient temperature at the midplane of the cavity near the cooled wall for various ξ

The figure 10 shows that in the beginning when the fluid motion is not strong, the heat transfer in the enclosure, is a dominated conduction and as the time increases and the convective motion in the enclosure becomes stronger, the heat transfer by convection becomes dominant. The overshooting phenomenon is clearly amplified or attenuated depending on the values selected for the dimensionless parameters characterizing the temperature-dependent viscosity. This phenomenon is often due to the corners effects, which are governed by the initial process of heat transfer. For small time, the heat is conducted into the fluid from the wall, the fluid near the heated wall tends to rise due to buoyant force. As time increases, the vertical flow becomes important near to the cavity corners, therefore, the levels of temperature decrease into the boundary layer. As time increases, the effect created near to the cavity corners is trained along the wall. This process is produced at different levels of the wall. The effect of corners cavity is considered as one-dimensional wave, which take place at the above extremity of the wall and propagates along the heated wall.

Thus, the results of figures 11 and 12 show the transient evolution of the temperature fields for $P_r=1$, $R_a=10^4$ and $\xi = \pm 1$; 0. Also, it can be seen that the transient temperature illustrates the same manifestations of different aspects of the overshoot phenomenon.

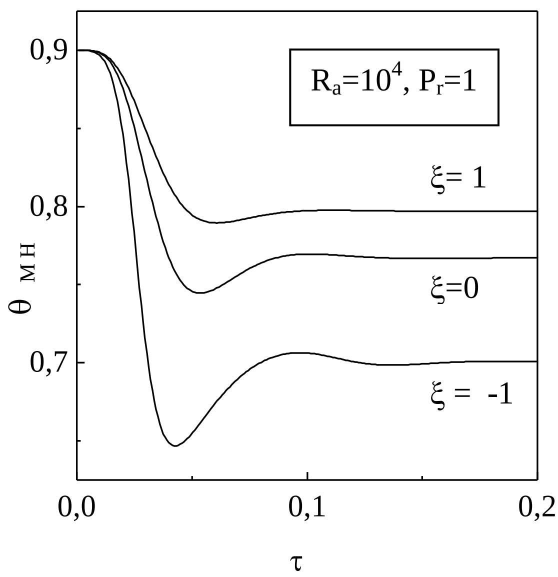


Figure 12: Transient temperature at the midplane of the cavity near the heated wall for various ξ

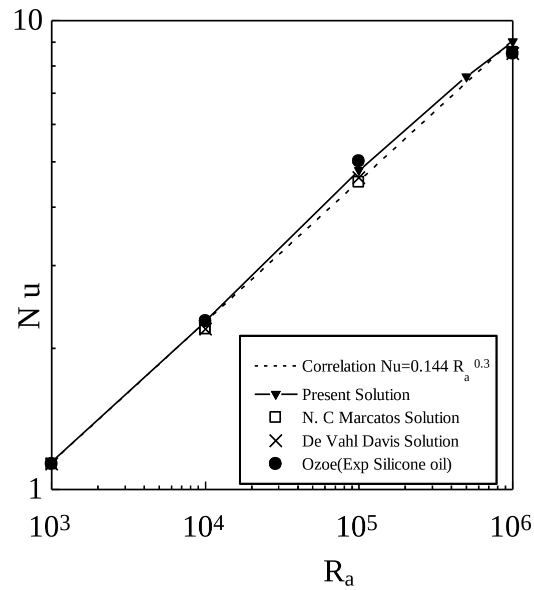


Figure 13: Variation of the average Nusselt number with respect to Rayleigh number

5.2 Heat transfer

The heat transfer rate across the system can be expressed in terms of Nusselt number. A local Nusselt number was integrated by a Simpson method. The average Nusselt number is defined as:

$$Nu = \int_0^h \left(\frac{\partial \theta}{\partial Y} \right)_{y=0} dX \tag{10}$$

Figure 13 shows the result of average Nusselt number for $P_r=1$ and $R_a = 10^4$. The numerical solution is compared with the finite analytic solution of Talaie [4] and the experimental measurement of Ozoe and al [23] in Silicone oil and numerical solution of Marcatos and Pericleous [24] and DeVahl Davis and Jones [25]. All results are in good agreement. This figure shows that the average of Nusselt number is approximated by a simple formula given as :

$$Nu = 0.144\, R_a^{\ 0.3} \qquad (11)$$

The results of figure 14 present the local Nusselt number $Nu_{X,Y=1}$ distribution along the right wall. It can be seen that the effect of viscosity variations increases, as the value of ξ becomes negative. The results of figure 15 show the transient evolution of the Nusselt number for $P_r = 1$, $R_a = 10^4$ and $\xi = \pm 1; 0$. It is found that the evolution illustrates the same flow regimes observed in the transient evolution of the vertical velocity and temperature, Nusselt number starts to increase first, pass trough a peak and then begins to decrease subsequently to the steady value. Thus, the values of viscosity contrast have a significant influence on heat transfer.

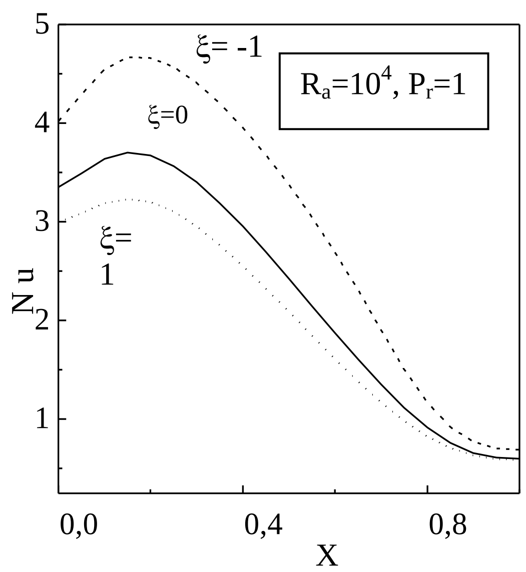


Figure 14: Nusselt number variation along the heated wall for various ξ

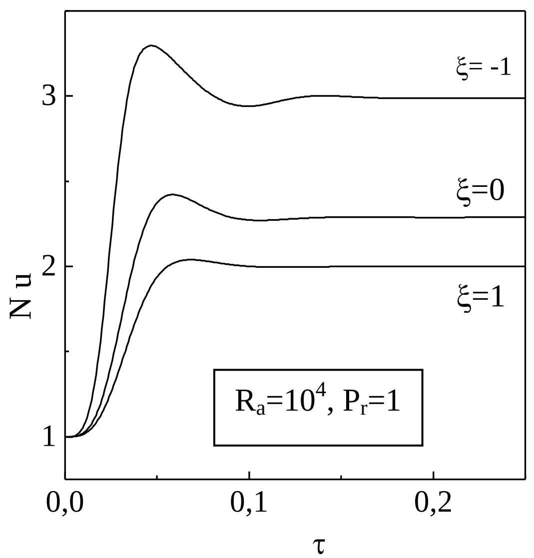


Figure 15: Transient evolution of Nusselt number for various ξ

6. CONCLUSION

A numerical method has been used to solve steady and transient natural convection with viscosity variation in two-dimensional square cavity. The temperature-dependent viscosity has a significant effect on flow patterns. The obtained results show that the effect of the dependence of viscosity on temperature for liquids can be quantitatively important in the study of a natural convection in a confined duct. The prediction of temperature and velocity fields is then extended to the enclosure for a range of Rayleigh number, Prandlt number and viscosity variation coefficient. This study shows that the temperature and velocity field rise to a peak and subsequently decrease to the steady value and this phenomenon is amplified or attenuated depending on the values selected for the dimensionless parameters characterizing the viscosity variation.

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