Component method for the assessment of the axial, shear and rotational stiffness of connections in old timber frames.

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I. INTRODUCTION

Wood is a natural material highly variable, susceptible to physical, chemical and biological deterioration. However, timber structures when they are properly designed and constructed have performed well for centuries. The fact that timber has been extensively used as a building material for centuries does not mean that we have a deep scientific understanding of its behavior. On the contrary, timber construction has from a very long time, to a large extent, been based on empirical experience and craftsmanship. The purpose of this work is to propose efficient and consistent solutions for the restoration of old timber structures.

I.1. OLD CARPENTRY JOINTS

In the field of structural engineering, informal languages commonly define a framework as a constructed assembly of joints separated by members. This definition sounds simple but the study of such structures is complex precisely because of many different types of joints used, particularly in old frames. There are hundreds of joint types however it remains possible to rank them into three major families:

- Lap joint: describes a technique for joining two pieces by overlapping them. A lap may be a full lap or half lap.
- Mortise and tenon joint: although there are many variations on the theme, the basic idea is that the end of one of the members (the tenon) is inserted into a hole cut in the other member (the mortise). The joint may eventually be pegged or wedged to lock it in place. Pegs are wooden pins while the term dowel usually refers to a steel pin.
- Cogging joint: kind of tenon on the end of a joist, received into a notch in a bearing timber and resting flush with its upper surface.

Those joints rely on the compression internal forces to keep facing surfaces in close contact and seldom on metal fasteners. Mortise and tenon joints can adequately carry shear and compressive through direct bearing of wood against wood. Under particular loading conditions, the joint may experience tensile loads that attempt to pull the tenon out of the mortise. In that situation, loads must be transferred between the mortise and tenon through pegs.

I.2 SEMI-RIGID ANALYSIS OF TIMBER FRAMEWORKS

Interest for the understanding of the behavior of joints is not really a new purpose. Because of the large utilization of steel, literature relating to steel connections is profuse, whereas for timber structures it is still much more marginal. Major factors influencing a static analysis of a structure depend on the structure itself (statically determinate or indeterminate) and the number and types of material used. Except for the determination of internal forces of statically determinate structures, stiffness properties of both members and joints are important. Because of a lack of background information on traditional timber joints, design usually assumes an ideally pinned or rigid behavior, neglecting this way the influence of the joint stiffness on the deflections or on the load redistribution at the ultimate limit state (ULS). However, the real semi-rigid behavior of the structure is somewhere between these two simplified models.

For a practical study of old timber frameworks according to present requirements, quick and efficient technique is needed. For that reasons, the component method used for the assessment of the rotational, axial and shear stiffness is easily automatable [1]. As pegged mortise and tenon joints are widespread joints used in most of old timber plane frames, our study essentially focused on them. Because of its asymmetric design, pegged mortise and tenon joint present different stiffness according to the sign of the applied load. The component method easily allows computing that stiffness. Some enhancements of the method are also proposed.

II. COMPONENT METHOD

Largely used in steel construction, the evaluation of the joint behavior consists of three main steps:

- Identification of the components.
- Evaluation of the mechanical properties of the components.
- Assembling active components to form a mechanical spring model.

Characteristics of the joint are determined by the basic components of the joint. The method allows characterizing all components of the joint independently of the type of loading applied to the joint which is of major interest. Each component is represented by an elastic stiffness coefficient only related to mechanical and geometrical data of the joint. One advantage of this method is that it can be easily applied for the characterization of the joint under combined loads.

II.1. ROTATIONAL STIFFNESS OF A PEGGED TENON AND MORTISE JOINT

Fig. 1 shows a bending moment (M) applied to a pegged tenon and mortise joint. Three couples of contact areas appear and transmit forces from one wooden element to the other. The first assumption consists in supposing that the peg imposes the position of the instantaneous center of rotation (ICR) of the joint. No friction is taken into account here. Let k a couple of surface i,j in contact. Total displacement $\delta_i$ in the normal direction to surfaces is:

$$
\delta_i = \delta_j + \delta_l = \frac{F_j}{k_j} + \frac{F_i}{k_i} = \frac{F_i}{k_k}
$$

where $F_k$ is the compression force acting on surfaces i and j, and $k_i$ and $k_j$ the respective contact stiffness.
Fig. 1. Tenon and mortice joints and the equivalent spring model when a positive bending moment is applied.

The equivalent spring constant of the stiffness \( k_i \) and \( k_j \) acting in series is simply:

\[
  k_{ij} = \frac{1}{\sum_{i,j} k_i}
\]

If \( M \) is the applied bending moment and \( \theta \) the relative rotation between the connected members, the rotational stiffness of the joint can be written as:

\[
  k_{\text{rot}} = \frac{M}{\theta} = \frac{\sum F_z z_i}{\theta} = \frac{\sum k_i z_i}{\theta} = \frac{\sum k_i (z_i \theta) z_i}{\theta} = \sum k_i z_i^2
\]

As the distances \( z_{ij} \) are simply defined when the ICR is known, the rotational stiffness only depends on the stiffness \( k_i \) of each couple of surface \( i,j \) in contact. This stiffness is obtained by means of elastic soils mechanics equations giving the settlements under a rectangular footing on a semi-infinite half space [1]:

\[
  k_i = \frac{E_{\alpha} \times b.h}{0.85}
\]

where \( b.h \) is the contact area and \( E_{\alpha} \) the MOE according to the direction \( \alpha \) to the grain. \( E_{\alpha} \) is estimated according to the Hankinson’s relation frequently used in timber engineering.

II.2. ENHANCEMENT OF THE STIFFNESS DEFINITION

The elastic definition of the stiffness given by Eq. 4 is obtained by means of Lambert and Whitman’s solution on basis of the theoretical deformation of an elastic half space [1]. For such a pegged tenon and mortise joint, surfaces in contact are rather small. Assumptions of an infinite half space are caught out and specific boundary conditions of free surface surrounding the contact area can not be neglected. To take this into account, a cut factor \( C_{m,E} \) is applied to the modulus of elasticity \( E_{\alpha} \). \( C_{m,E} \) is defined in Table 1 where \( \lambda \) is the slenderness of the contact area. This factor was defined with the help of elastic finite element models (FEM) using an orthotropic material, contact and different slenderness of contact areas [2].

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>CUT FACTOR ( C_{m,E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( C_{m,E} )</td>
</tr>
<tr>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>0.5005</td>
</tr>
<tr>
<td>3</td>
<td>0.4355</td>
</tr>
<tr>
<td>&gt;3</td>
<td>0.39</td>
</tr>
</tbody>
</table>

II.3. ENHANCEMENT OF THE ICR DEFINITION

In some experimentations on full-scale joints, failure occurred because of an excess of shearing in the peg or in the tenon. This specific failure mode argues that even if the peg imposes the position of the ICR at the beginning, this one should move during the loading. For the localization of the ICR, a geometrical research field is defined and an iterative process is implemented (Fig. 2).

For each point of this domain, let \( u, v, \theta \) its displacements and rotation. Firstly, contact areas are identified and with the help of the contact stiffness \( k_i \), contact forces are computed. Secondly, the equilibrium of the joint is written assuming that only a known bending moment is introduced in the joints. ICR is then supposed to be the point for which computed displacement \( \sqrt{u^2 + v^2} \) is minimal. As the peg is not the ICR, a load appears in the peg and the method needs the definition of the stiffness \( k_{peg} \) characterizing the contact between the peg and the peg hole. For this, tests in laboratory were performed. Because of orthotropic material, the stiffness was measured for a loading acting parallel and perpendicular to the grain. Average results are presented in Table 2.

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>AVERAGE STIFFNESS ( k_{peg} ) (BETWEEN THE PEG AND THE PEG HOLE).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>Perpendicular</td>
</tr>
<tr>
<td>( k_{peg} ) [N/mm]</td>
<td>3909 (COV 16%)</td>
</tr>
<tr>
<td>Ultimate load [N]</td>
<td>7010 (COV 5%)</td>
</tr>
</tbody>
</table>

Some results presenting the computed position of the ICR and the associated rotational stiffness of the joint under a clockwise moment \( M^+ \) are presented on Fig. 3.

Fig. 2. Research of the ICR of the joint.

Fig. 3. Positions of the ICR and associated rotational stiffness.
II.4. AXIAL AND SHEAR STIFFNESS

The axial and shear stiffness of each joint is simply defined following a similar approach. Contact areas are identified according to the direction of the load (Fig. 4), stiffness is associated to each couple of surfaces in contact including the peg and finally the total stiffness is computed automatically.

This enhanced method was programmed under MATLAB environment. A parametric approach allows easily computing the axial, shear or rotational stiffness of any pegged tenon and mortise joint under any kind of solicitation.

III. EXAMPLE OF APPLICATION

The fortified castle of Ecaussines-Lalaing is situated in Belgium in the south part of the country. Built during the 12th century, the castle is composed of three wings surrounded by ditches, a court-yard, fortifications and a gothic tower. The frames, simply posed on masonry walls, are made of one large tie-beam and one or two high collars tying opposing rafters and stiffening the roof. Loads applied (wind, snow, dead load and service loads) are evaluated according to Eurocode ULS and SLS combinations.

Fig. 5 presents results of the complete semi-rigid model (shear, axial and rotational stiffness) and compares them with hinged model. Bending moments had increased in rafters and collar (20%) which is of course of a major importance for the ULS diagnosis. Braces undergo an important decrease (>100%) of the maximum tensile load. This reduction is also beneficial for the design as it is difficult to guarantee the strength of such old carpentry joints in case of important tensile loads.

IV. CONCLUSION

Historic timber joints are numerous and complex. For their fiddly and efficient study, the component method gives quick and handy results. However, because of initial crude assumptions for the assessment of contact stiffness, an enhancement of the method was proposed. Firstly, a cut factor $C_{m,E}$ is applied to the timber modulus of elasticity $E_a$ for the assessment of the contact stiffness. Secondly, for the definition of the rotational stiffness, accordingly to full-scales tests, we assumed that the peg does not imposes the position of the instantaneous center of rotation. Its position is computed solving the equilibrium equations of the joints.

For comparison, finite elements models and experimentations were performed to validate the proposed enhancements of the component method. The component method seems slightly overestimating the stiffness of the joint about 40% which actually is satisfactory for a practical use on frameworks. Some improvements of the method are still in progress:

- about the unrealistic stress distribution appearing on contact areas (actually supposed uniform)
- about the definition of the ultimate load of the joint
- about the modeling of the joint behavior with a bi-linear model. This model would allow to introduce unavoidable initial gap in the model.

Finally, an application of the proposed method to the study of old timber frameworks has been performed. Results show that the joint stiffness is an important parameter for the response of the structure. However, the introduction in models of the rotational stiffness alone is not enough. Both axial and rotational stiffness have to be introduced in finite elements models for a fiddly study. This semi-rigid study allowed to propose some reinforcements of joints but also to answer major questions asked by restorers about the eventuality to remove some invasive reinforcements added in the last decades.

REFERENCES