Contact model for a rough surface exhibiting fractal behaviour

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Introduction
Surface roughness is recognized as an important factor for the operational behaviour of machine elements. This factor is important because it influences the dry or lubricated contacts, friction and wear phenomena or surface failure. The problem of elastic contact of rough surfaces was studied from many points of view, but a numerical treatment in a contact zone requires detailed discretization of actual topography [1]. There are some specific problems in discussion: (a)- estimation of a reasonable portion of the assembly of bodies in contact; (b)- the need to make the model as manageable as possible without introduction of realistic simplifications; (c)- to obtain a good estimation of the forces that are transferred in a contact zone; (d)- the connection between the shape of the roughness profile, pressure distribution and the subsurface stress field.

The main target in this paper is the development of a contact model which includes surface roughness and to identify subsequent surface stress field in comparative rough model contacts.

Contact models
Regarding the models for real contact area there are various solution proposals. The first problem is the geometry of the asperities: some of these solutions transform the real rough Hertzian contact model. First of all we consider a real profile (2D model) for contact. At this scale a profilometer was used to measure the profile from an arbitrary machined metal sample of a hobbled gear.

This model incorporates the physical model of contact between one cylinder with a given geometry (radius R) and a plane.

We consider the model between a smooth plane and a cylinder with a composite roughness y(x). We assume that the deformations of the asperities are elastic. The deformations of the model are: (a) – deformations of the asperities at a micro scale; (b) – deformation of the elastic body. The deformation of the rough surface is shown in fig.1:

\[
d(x) = y(x) + y_p - \left(\frac{x}{2R} + y_0(x)\right)
\]

where \(d(x)\) is the deformation of the roughness profile, \(y_0(x)\) is the roughness height measured from the mean line, \(y(x)\) is the distance of the deformed plan from the referenced plane, \(y_p\) is the composite approach distance and \(x^2/(2R)\) is the distance of the point on the circle to the rigid plan.

In equation (1) \(d(x)\) represent only the elastic or plastic deformation of the asperity. If the \(y_p\) approach is given, the macro geometry is known (the circle) and the distribution of the asperities could be determinate \((y_0(x))\), the deformation \(d(x)\) is a function of distance \(y(x)\). So, if we find an aproach we can calculate the deformation. In this study we will consider the approach as the deformation of a single reference asperity in contact with a smooth plane.

If we consider a different way to define (to acquire) a geometry of the model we have to introduce another assumption.

In the literature some specialists suppose that the surface is in a smooth equivalent one with asperities, others made the assumption that the asperities are different known geometrical models such spheres, Greenwood and Williamson [2], Greenwood and Tripp [3], elliptical paraboloids, Bush et al. [4], pyramidal forms et al.

So in this study we will consider the geometry of a sphere respective a semi-circle asperity profile. The second problem concerns the distribution of the “asperities”.

There is a model based on the concept of deterministic distributions of asperities, Archard, [5], or a model based on a statistical distribution as in [3], [6]. The later development use a random process theory as in Whithouse and Archard [5], Bush et al. [4], Nayak [7].

To overcome the dependence of traditional roughness parameters on sample size the surface topography was described by fractal geometry (Yan and Komvopoulos [8]).

In equation (2) is calculated the function \(y(x)\) of the two-dimensional fractal surface used in this paper.

\[
y(x) = \left(\frac{q}{1}\right)^{(D-n)} \sum_{q=1}^{\infty} \frac{\cos(2\pi x q^n)}{q^{2-D/n}}
\]
In this equation \( x \) is the dimension along fractal sample length, \( l \) is the fractal sample length, \( q \) is a frequency parameter concerning the fractal independent design, \( D \) is the fractal control dimension (controls the contribution of high and low frequency components in the surface function), \( \gamma \) is a scaling parameter and \( n \) is a frequency index as an integer number given by the relation:

\[
n_{\text{max}} = \log \frac{1}{l_{\text{tot}}} \log \gamma
\]  

(3)

where \( l_{\text{tot}} \) is the total length of the sample.

Concerning some aspects of this model we suppose that the value for \( D \) is 1.5 (2 is a maximum value for a smooth surface), the value for \( \gamma \) is 1.5 (typical value). Concerning the value for \( n_{\text{max}} \) we found in a short analysis that the maximum value, for specific parameters as data given, is less than 4. For a value of fractal sample length between (1-10) % of the total cutoff length we found the result for \( y(x) \) function presented in fig. 2:

![Fig. 2: The function of two dimensional fractal surface](image)

**Finite element model**

With these assumptions we are going further in order to accomplish the contact model. We now are concerns with a quasi-static indentation (or elastic deformation) model. For this purpose we consider a model with only one single asperity in contact with an elastic medium.

We use the software DS Solidworks 2007 to create the parts of the contact assembly and the DS CosmosWorks 2007 software for the analysis. The first model created is presented in fig. 3:

![Fig. 3: Finite element model of contact](image)

Using the results of the FEM study we now have values for the approach \( y_p \); also we consider the distribution of the asperities given by fractal model and we can calculate the deformations \( d(x) \).

**Pressure distribution and stress analysis**

In the frame of this scale contact model, using a semi-infinite body subjected to pressure given by a rough profile, load along the discontinuous contact line is taken into account:

\[
d(x) = \frac{2(1 - \nu^2)}{\pi E} \int_{l_a}^{l_b} \frac{1 - x}{1 - x_i} \rho(x) d(x)\]

(4)

Equation (4) allow the determination of the pressure distribution for a roughness profile if we know all the geometrical data and the material properties. In this equation we consider the sample length \( l \) as a sum of discontinuous contacts in this interval \([l_a, l_b] \). According to...
hypothesis of this study we consider the asperities will deform elastically. Also, in this moment we can find the elastic deformation due to elastic contact. Suppose that there are two asperities in contact with elastic body.

In the figure 5 is shown the result of the stress distribution due to non-uniform distributed pressure concerning this hypothesis and using the equations (1), (2) and (4)

![Fig. 5: Von Misses stress field distribution for two asperities profile](image)

This result obtained using the equation (4) is compared with the result obtained with FEM in the frame of the same model as is shown in figure 3. In the model presented in figure 3 the distributed force along the top edge is uniform distributed.

The result is shown in figure 6. According figure 5 and figure 6 we can see a qualitative correlation concerning the von Misses stress. We can see the von Misses stress distribution in the region of one asperity in contact with a smooth plane for the model of a profile with two identical asperities distributed along the “sample” length. In both figures we can observe a slight asymmetry in the stress distribution and the explanation is given by the influence of the second asperity of the profile in contact in the neighbourhood. Also, there is a good correlation between the stress numerical values obtained with these two models.

![Fig. 6: Von Misses stress field distribution for two asperities profile using FEM analysis](image)

We consider that the model presented in the equations (1)-(4) using the data provided by the FEM CosmosWorks could be used in the next study for the stress analysis of a layerd elastic solid in contact with a rough surface.

**Conclusions**

A contact method and model was presented for an elastic half-space in contact with a rough surface exhibiting self-affine or fractal behaviour. This model is based of elastic deformation of one asperity in contact with elastic space, of the fractal asperities distribution and the data provided using the finite element method.

Results of the contact pressure distribution and subsurface stresses for an example using a profile with two asperities show the power of the method.

A good agreement with a model using only the Finite Element Method also show the power of the method and model presented.

**References**