Effect of rotation on acoustic waves propagating in a thermo-piezoelectric material half space

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1 Abstract

The analysis of the rotation effect on the characteristics of the acoustic waves propagating in a half space thermopiezoelectric materials surfaces in the framework of linear thermo-piezoelectricity including Coriolis and centrifugal forces is presented. It shows that the Rayleigh and Bleustein-Gulyaev waves, depending of thermal and physical properties of materials, may be suppressed by rotation, and that these surface waves, if they exist, are generally dispersive. The effects of the Coriolis and centrifugal forces on dispersion are generally of the same order. The influence of thermo-mechanical and pyroelectric couplings has not to be neglegected.

2 Introduction

Thermo-piezoelectric materials represent one of popular candidate materials being considered for use in a the performance of existing aerospace structures. In general, thermo-piezoelectric materials provide fast response times, good dynamic behavior, the capability to be used as either sensors or actuators, simple integration into a structure, low power requirement, a readily obtainable commercial supply, and long familiarity through previous applications in transducers. Thermal effects greatly influence the performance of piezoelectric actuators and sensors, especially when they are required to operate in severe temperature environments. The governing equations of a thermo-piezoelectric material where the mechanical, electrical, and thermal fields are coupled have been derived by Mindlin (1974). General theorems of thermo-piezoelectricity are given in Nowacki (1978) and Iesan (1989). Huang and Batra (1996) and Yang (1998) developed equations governing deformations of piezothermoviscoelectric materials. Tauchert (1992) and Jonnalagadda et al. (1994) developed plate theories for thermopiezoelectric laminated plates. Tang et al. (1996) assessed the accuracy of various thermopiezoelectric plate models. Lee and Saravanos (1996) developed a coupled, layerwise theory to analyze the thermopiezoelectric behavior of composite structures. Ishihara and Noda (2000) studied thermopiezoelectric laminates including the effects of transverse shear deformation and coupling. Due to the coupling of the mechanical, electrical, and thermal fields and its inherent anisotropy, the analysis of thermopiezoelectric materials is a challenging task and relatively few analytical solutions to the three-dimensional governing equations are available in the literature. Characteristics of waves propagating in solids and their dependence upon various geometric and physical parameters have been under continued study. The effect of pre-stress, acceleration, and temperature variation, etc., on wave speed or frequency provides the foundation for the development of many acoustic sensors (White, 1998). Particularly, frequency shifts due to rotation have been used to make gyroscopes, i.e., angular rate sensors, see Tiersten et al. (1980, 1981), Lao (1980), Clarke and Burdess (1994), and Yang et al. (1988), or surface waves over a half space (Tiersten et al., 1980; Lao, 1980; Clarke and Burdess, 1994). The present study focuses on the rotation effect upon waves propagating in the half space thermo-piezoelectric surface. There is a little literature on analysis of the rotation-disturbed acoustic waves propagating along the surface of a thermopiezoelectic half-space. our analysis shows that these effects on the surface acoustic waves speed are of the same order. Our discussion will be focused on two types of surface waves, i.e., the Rayleigh wave (Rayleigh, 1885), and the Bleustein-Gulyaev wave (Bleustein, 1968, Gulyaev, 1969). To illustrate some numerical results, three polycristalline thermo-piezoelectric ceramics are chosen, i.e., PZT-5H, PZT-6B, and BaTiO3. Our analysis suggests the presence of these waves in the absence of rotation. However, when the solid rotates slowly, we note that they are dispersive.

3 Mathematical Formulation

The current formulation is based on the linear theory of thermo-piezoelectricity in which the equations of linear elasticity are coupled to the charge equations of electrostatics through the piezoelectric constants and to the temperature equation through the thermo-mechanical and pyroelectric coefficients. Due to the assumption of a linear theory, both mechanical and electric body forces and couples are neglected in the derivation. The model assumes infinitesimal deformations and strains. We consider a linear thermo-piezoelectric body that occupies the half space, denoted by $x_2 \leq 0$ in the Cartesian frame. To study the rotation effect on the phase velocity of surfacic wave propagating on a half space of poly-crystalline piezoelectric ceramics, we let the body rotates about the x_2-axis at a constant rate Ω. Introducing the effects of force of coriolis and centrifugal force, the equation of motion and Coulomb's laws are given by

$$T_{ij} + f_i - (2\Omega j \times \dot{u}) - (\rho\Omega^2 j \times (j \times u)) = \rho\ddot{u}_i, D_{i,i} = 0, E = -\nabla\Phi \tag{3.1}$$

where T_{ij} is the symmetric stress tensor, f_i represents the mechanical body force, D_i is the electric displacement and Φ the electric potential. The heat and linear thermo-piezoelectricity Constitutive equations are

$$T_{ij} = c_{ijkl}\gamma_{kl} - e^T_{kij} - \lambda_{ij}\theta, D_m = e_{mkl}\gamma_{kl} + \epsilon_{mk}E_k + p_i\theta, (\rho c_0/\theta_0)\dot{\theta} + \lambda_{ij}\dot{\gamma}_{ij} + p_i\dot{E}_i = -\kappa_{ij}\theta_{,ij} \tag{3.2}$$

Here E is the electric vector field, D the dielectric displacement, θ the temperature, γ the linearized strain tensor, θ is the temperature difference, and θ_0 is the initial temperature. The matrices c, e, λ, ϵ, κ and p are respectively, elastic, piezoelectric, thermal, dielectric, thermal conductivities, and pyroelectric symmetric tensors. c_0 is the specific heat per unit mass. We are interested in waves propagating along a surface that is electroded and free of mechanical loads. Introducing a new Bluestein variable defined by $\Phi = \Psi + (e_{15}/\epsilon_{11})u_3$, for $x_2 = 0$ the following boundary conditions are considered

$$T_{12}, T_{22}, T_{32} = 0, \Phi = 0, \theta = 0 \tag{3.3}$$

To seek surface wave's solutions, we require that both the displacement and electric potential diminish with depth, i.e.

$$\lim_{x_2 \to -\infty} (u, \Phi, \theta) = 0 \tag{3.4}$$

Considering a wave propagating along the x_1-direction, and for the upper half-space, the solutions can be written as

$$u_{1,2,3} = A_{1,2,3}e^{k\eta x_2}e^{ik(x_1 - vt)}, \theta = A_4 e^{k\eta x_2}e^{ik(x_1 - vt)}, \Psi = iA_5 e^{kx_2}e^{ik(x_1 - vt)} \tag{3.5}$$

4 Surface waves solutions

Substituting the solutions (3.5) into equations (3.1) leads to the following complex eigenvalue problem

$$a_{ij}A_j = 0 \tag{4.1}$$

where $a_{11} = v_2^2\eta^2 - v_1^2 + v^2(1 + (\Omega/\omega)^2)$, $a_{12} = a_{21} = i\eta(v_1^2 - v_2^2)$, $a_{14} = i\alpha$, $a_{22} = v_1^2\eta^2 - v_2^2 + v^2$, $a_{23} = 0$, $a_{24} = i\alpha\eta$, $a_{33} = v_3^2(\eta^2 - 1) + v^2(1 + (\Omega/\omega)^2)$, $a_{31} = -a_{13} = 2iv^2(\frac{\Omega}{\omega})$, $a_{32} = 0$, $a_{14} = -\alpha$, $a_{24} = i\alpha\eta$, $a_{34} = 0$, and $a_{44} = i\beta_0 + \beta$

with $\alpha = -\lambda_{11}/k\rho$, $\beta_0 = \rho c_0/\theta_0 k^2$, $\beta = \kappa(\eta^2 - 1)/kv$, $v_1^2 = c_{11}/\rho$, $v_2^2 = c_{66}/\rho$, $v_3^2 = \bar{c}_{44}/\rho$, $c_{66} = (c_{11} - c_{12})/2$, $\bar{c}_{44} = c_{44} - (e_{15}^2/\epsilon_{11})$

Here Ω/ω is the rotation ratio between the rotation of body and the frequency and A_j are the eigenvectors corresponding to the eigenvalues η^2. For (4.1) to have non-zero solutions, the determinant of the coefficient matrix $A = a_{ij}$ must vanish and this leads to an complex polynomial equation of degree eight for η_j. This equation indicates that the frequency ω and the phase velocity v are generally dependents unless $\Omega = 0$, i.e., the surface waves are generally dispersive due to the rotation effect. The Coriolis and centrifugal forces are of the same order in terms of the ratio $\frac{\Omega}{\omega}$ which a small parameter and they depend upon thermal and physical material properties. We assume now that the eqs (4.1) has four distinct complex roots for η_j^2. This leads to the following representation for the wave solutions

$$(4.2)\qquad u_{1,2,3} = \sum_{j=1}^{4} C_j A_{1,2,3}^{(j)} e^{k\eta_j x_2} e^{ik(x_1 - vt)}, \theta = \sum_{j=1}^{4} C_j A_4^{(j)} e^{k\eta_j x_2} e^{ik(x_1 - vt)}, \Psi = iC_5 e^{kx_2} e^{ik(x_1 - vt)}$$

Where A^j $(j = 1, .., 4)$ are the eigenvectors corresponding to the eigenvalues η_j^2. The complex magnitudes C_j and the real magnitude C_5 are to be determined. Substituting (4.4))into the boundary conditions $(3.3)_{1,2,3}$ yields to the following eigenvalue equation

$$(4.3)\qquad b_{ij} C_j = 0$$

with $b_{1j} = \eta_j A_1^{(j)} + iA_2^{(j)}, b_{15} = 0$, $b_{2j} = ic_{12} A_1^{(j)} + c_{11}\eta_j A_2^{(j)} - \frac{\lambda_{11}}{k} A_4^{(j)}$,$b_{25} = 0$, $b_{3j} = \bar{c}_{44}\eta_j A_3^{(j)}, b_{35} = e_{15}$, $b_{4j} = e_{15} A_{(3)}^{(j)}$, $b_{45} = \epsilon_{11}$, $b_{51} = A_4^1, b_{52} = 0$, $b_{53} = 0$, $b_{54} = A_4^4$, $b_{55} = 0$, $(j = 1, .., 4)$

For eq.(4.3) to have non-zero solutions, the determinant of complex coefficients matrix must vanish. The components of the above complex determinant have a polynomials forms, and they depend in v and k. Therefor to vanish the eq.(4.13) and determinate the phase velocity v, the real and/or imaginary parts must vanish. To determine the dependence of the surface wave speed upon the rotation and plot the curves, we let the rotation rate increase gradually from zero and solve eqs.(4.1) and (4.3) using iteration procedure. For a given real value of k, we choose a real trial v_0, we determine the eigenvalues, the four roots of η_j^2 of the polynomial equation (4.1), solve (4.3) for the surface wave speed v and repeat these steeps until a prescribed is met $|v_{i+1} - v_i| < \epsilon << 1$. Substituting the so-determined η_j^2 and their corresponding eigenvectors results in a formally complex determinant, or equivalently two real determinants (the real and imaginary parts of the complex determinant) returns to the determination of v. The so-determined v together with the given Ω/ω defines a dispersion curve.

5 Conclusion

Our analysis shows that the Rayleigh and Bleustein-Gulyaev waves are generally dispersive under effect of rotation. One notices the existence of certain abnormal dispersion for small rotation ratios, i.e. a wave velocity increasing locally when the rotation ratio increase. The effects of Coriolis and centrifugal forces are of the same order. The simulations for the three chosen thermo-piezoelectric crystals, are shown that the themo-mechanical and pyroelectric couplings influence, only, on the Rayleigh wave dispersion.

6 References

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