Fast parametrically excited van der Pol oscillator with time delay state feedback

Mohamed Belhaq and Simohamed Sah
Laboratory of Mechanics, University Hassan II-Aïn Chock
PB 5366 Maârif, Casablanca, Morocco
e-mail: mbelhaq@yahoo.fr
e-mail: simohamedsah@yahoo.com

Abstract
We investigate the interaction effect of fast vertical parametric excitation and time delay state feedback on self-excited vibrations (limit cycle) in a van der Pol oscillator. We use the method of direct partition of motion to derive the main autonomous equation governing the slow dynamic in the vicinity of the trivial equilibrium. Then we apply the multiple scales method on this slow dynamic to derive a second order slow flow system describing the modulation of slow dynamic. In particular we analyze the slow flow to analytically approximate regions in parameter space where self-excited vibrations can be eliminated. It was shown that fast vertical parametric excitation, in the presence of time delay state feedback, can suppress self-excited vibrations for an appropriate choice of feedback gains, time delay and the excitation frequency.

1 Introduction

In this paper, we study the interaction effect of a vertical fast harmonic (FH) parametric excitation and time delay state feedback on self-excited vibration in a van der Pol oscillator. Great attention has been paid in the last decade to the study of nontrivial effects of FH excitation on mechanical systems. Belkhman [1] developed a technique called the direct partition of motion based on splitting the dynamic into fast and slow motions. This method provides an approximation for the small fast dynamic and the main equation governing the averaged slow dynamic. A number of studies used this mathematical tool to analyze properties and dynamics of some mechanical systems. Thomsen [2] analyzed some general effects of strong high-frequency excitation.
In the present work we investigate the effect of vertical high-frequency excitation on self-excited vibrations in a van der Pol oscillator under time delay state feedback, involving both displacement and velocity feedback.
Note that delayed parametrically excited oscillations has been considered by Insperger and Stepan [3]. They studied the stability chart for the delayed linear Mathieu equation using the method of exponential multipliers. Morrison and Rand [4] investigated the dynamic of the delayed nonlinear Mathieu equation in the neighbourhood of 2:1 resonance. Hu et al. [5] considered the primary resonance and the 1/3 subharmonic resonance of a forced Duffing oscillator with time delay state feedback. Using the multiple scale method [6], they demonstrated that appropriate choices of feedback gains and time delay are possible for a better vibration control.
To analyse the effect of vertical FH parametric excitation and time delay state feedback on the suppression of limit cycle in van der Pol oscillator, we apply, in a first step, the method of direct partition of motion to derive the main autonomous equation governing the slow dynamic of the oscillator. The multiple scales method is then performed on the slow dynamic to obtain a second-order slow flow. The analysis of this slow flow provides analytical approximations of regions in parameter space where self-excited vibrations can be eliminated.

2 Slow dynamic

Small vibrations around the origin of pendulum under time delay state feedback and with point of suspension subjected to a vertical parametric forcing and to a self-excitation can be described in non-dimensional form by the following equation

$$\begin{aligned}\frac{d^2x}{dt^2} &- (\alpha-\beta x^2)\,\frac{dx}{dt}+x-\delta x^3 = a\Omega^2\,x\,cos\,\Omega t\\ &+ k_1\,x(t-T)+k_2\,\frac{dx(t-T)}{dt}\end{aligned} \tag{1}$$

where the parameters α and β are assumed to be small, a is the excitation amplitude, Ω is the parametric excitation frequency, and the parameters k_1, k_2 and T are the displacement gain, the velocity gain and the delay period, respectively.
The goal here is to investigate the interaction effect of vertical high-frequency excitation Ω and delay parameters k_1, k_2 and T on the control of self-excited vibrations that can take place in system (1). Applying the Direct Partition of motion on Eq. (1) and setting $\delta = 0$ since δ has no effect on the stability of the origin

yields

$$D_1^2 z \quad - \quad (\alpha - \beta z^2) D_1 z + (1 + \frac{(a\Omega)^2}{2}) z$$
$$= \quad k_1\, z(T_1 - T) + k_2\, D_1 z(T_1 - T) \quad (2)$$

3 Multiple scales and slow flow

We examine the effect of high-frequency excitation on the limit cycle in the slow dynamic (2) by performing the multiple scales method [6]. Introducing a small bookkepping parameter μ, scaling $\alpha = \mu\alpha$, $\beta = \mu\beta$, $k_1 = \mu k_1$ and $k_2 = \mu^2 k_2$ and assuming that α, β, k_1 and k_2 are positive and taking the form of the solution of the zero-order problem as

$$z_0 = R(\tau_1, \tau_2)\, e^{i\,(\omega_0\,\tau_0 + \phi(\tau_1,\tau_2))} + cc, \qquad (3)$$

we obtain up to the second-order the slow flow modulation equations of amplitude and phase

$$\dot{R} \quad = \quad (\frac{\alpha}{2} - \frac{k_1}{2\omega_0}\, sin\,\omega_0 T + \frac{k_2}{2}\, cos\,\omega_0 T$$
$$- \quad \frac{k_1^2}{8\omega_0^3}\, sin\, 2\omega_0 T)\, R - (\,\frac{\beta}{8} + \frac{\beta k_1}{8\omega_0^2}\,)\, R^3 \quad (4)$$

$$\dot{\phi} \quad = \quad -\frac{\lambda}{2\omega_0}\, cos\,\omega_0 T - \frac{k_2}{2}\, sin\,\omega_0 T - \frac{\alpha^2}{8\omega_0}$$
$$- \quad \frac{k_1^2}{8\omega_0^3}\, cos\, 2\omega T + (\frac{16\beta k_1}{\omega_2}\, sin\,\omega_0 T - \frac{\beta\alpha}{4\omega_0})\, R^2$$
$$+ \quad \frac{19\beta^2}{256\omega_0}\, R^4 \qquad (5)$$

4 Equilibria and self-excited oscillations

Non-trivial equilibria, obtained by setting $\dot{R} = \dot{\phi} = 0$ in Eqs. (4)-(5), are given by

$$R = 2\sqrt{\frac{(\alpha - \frac{k_1}{\omega_0}\, sin\,\omega_0 T + k_2\, cos\,\omega_0 T - \frac{k_1^2}{4\omega_0}\, sin\, 2\omega_0 T)}{\beta\,(1 + \frac{k_1}{\omega_0^2}\, cos\,\omega T)}} \qquad (6)$$

Then for the existence of limit cycle the non-trivial equilibrium in Eq. (6) must be real which leads to the condition

$$\alpha - \frac{k_1}{\omega_0}\, sin\,\omega_0 T + k_2\, cos\,\omega_0 T - \frac{k_1^2}{4\omega_0}\, sin\, 2\omega_0 T \;\geq\; 0 \qquad (7)$$

Note that the term ($1 + \frac{k_1}{\omega_0^2}\, cos\,\omega_0 T$) appearing in the denominator of the amplitude R is positive.
On the other hand, the frequency of the limit cycle is given by

$$\omega = \omega_0 + \frac{d\phi}{dt} \qquad (8)$$

A condition for the existence of the limit cycle is satisfied when the frequency ω is positive which is guaranteed in Eq. (8).

In Fig. 1 analytical approximations based on Eq. (7) are compared to numerical results obtained by integrating Eq. (1) in the (T, Ω) plane. This Figure suggests that for fixed feedback gains k_1 and k_2, there exists a maximal value of the frequency excitation Ω under which the limit cycle can be eliminated. This value is given by the relation

$$\Omega_{max} = \frac{1}{a}\sqrt{2(\frac{k_1^2}{\alpha^2 - k_2^2} - 1)} \qquad (9)$$

provided that $k_1^2 + k_2^2 \;>\; \alpha^2$ and $k_2 \;<\; \alpha$. In the case where $k_1^2 + k_2^2 \;<\; \alpha^2$, the suppression of the limit cycle cannot be realized. The corresponding time delay to Ω_{max} is

$$T_{max} = \frac{\sqrt{\alpha^2 - k_2^2}}{k_1}\,(\pi - arctan(\frac{\sqrt{\alpha^2 - k_2^2}}{k_2})) \quad (10)$$

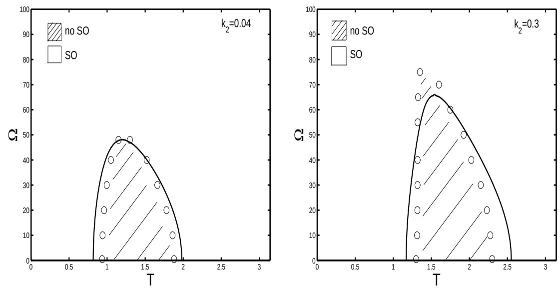

Figure 1: Comparison between analytical results (solid line) based on Eq. (7) and numerical integration (circles) of the original system, Eq. (1); $a = 0.02$, $\alpha = \beta = 0.5$, $\delta = 1/6$, $k_1 = 0.6$. SO : self-oscillations.

From Fig. 2 we remark that the region where the limit cycle can be eliminated in the (T, k_2) plane exists for all values of the velocity feedback k_2 in certain range of the excitation frequency Ω and in the neighborhood of $T_c = \frac{\pi}{2\omega_0}$. Nevertheless, this region splits into two regions as we exceed the critical value of the excitation frequency

$$\Omega_c = \frac{1}{a}\sqrt{2(\frac{k_1^2}{\alpha^2} - 1)} \qquad (11)$$

corresponding to $\frac{dk_2}{dT} = 0$, and as long as we increase Ω these two resulting regions decrease.

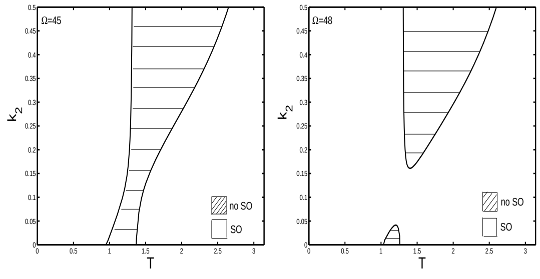

Figure 2: Curves delimiting the suppression regions of limit cycle, Eq. (7), for different values of Ω; $a = 0.02$, $\alpha = \beta = 0.5$, $k_1 = 0.6$. SO : self-oscillations.

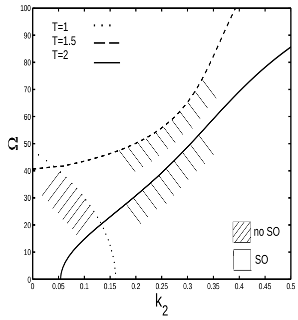

Figure 3: Curves delimiting the suppression regions of limit cycle, Eq. (7), for different values of T; $a = 0.02$, $\alpha = \beta = 0.5$, $k_1 = 0.6$. SO : self-oscillations.

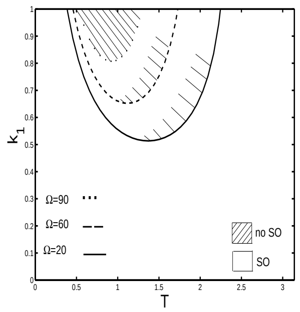

Figure 4: Curves delimiting the suppression regions of limit cycle, Eq. (7), for different values of Ω; $a = 0.02$, $\alpha = \beta = 0.5$, $k_2 = 0.04$. SO : self-oscillations.

In Fig. 3, we can see how the region of non existence of the limit cycle changes as we approach $T_c = \frac{\pi}{2\omega_0}$ from both below and above. Fig. 4 shows the region in the (T, k_1) plane where the limit cycle can be eliminated. This can take place only after a certain value of the displacement feedback given by

$$k_{1\,min} = \sqrt{1 + \frac{(a\Omega)^2}{2}}\sqrt{\alpha^2 - k_2^2} \qquad (12)$$

Furthermore, this region decreases by increasing Ω. In Fig. 5 we illustrate the time series of the original system Eq. (1). These figures depict the birth of a self-excited vibration by changing the frequency Ω for fixed delay parameters.

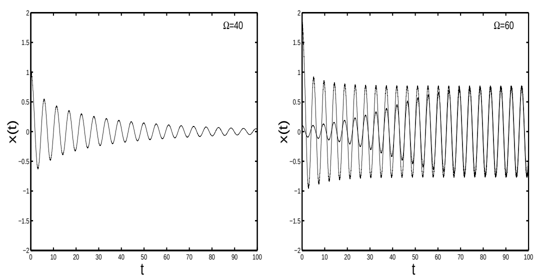

Figure 5: Time series for the original system, Eq. (1), with parameter values as for Fig. 1; $\delta = 0$, $k_2 = 0.04$, $T = 1.4$.

5 Conclusions

In this work we have investigated the interaction effect of vertical FH parametric excitation and time delay state feedback on self-excited vibrations in a van der Pol pendulum. We have shown that vertical FH parametric excitation can eliminate undesirable self-excited vibrations by choosing appropriate feedback gains and time delay. Even for low values of the parametric frequency Ω, this suppression can be realized in the vicinity of the time delay T_{max}, Fig. 1. Note that these self-excited vibrations persists in the absence of time delay state feedback as shown in [7].
It is worth noting that for certain range of excitation frequency Ω in the vicinity of time delay $T_c = \frac{\pi}{2\omega_0}$, self-excited vibrations can be eliminated for all values of the velocity feedback gain k_2, Fig. 2, left. In contrast, this elimination can be achieved only if the displacement feedback gain k_1 reaches certain values, Fig. 4.

References

[1] I.I. Blekhman, Vibrational Mechanics - Nonlinear Dynamic Effects, General Approach, Application, World Scientific, Singapore, 2000.

[2] J.J. Thomsen, Some general effects of strong high-frequency excitation: stiffening, biasing, and smoothing, J. Sound Vibrat. 253 (2002) 807-831.

[3] T. Insperger, G. Stepan, Stability chart for the delayed Mathieu equation, Proceedings of the Royal Society, Mathematical, Physical and Engineering Sciences, 2002,pp. 1989-1998.

[4] T.M. Morrison, R.H. Rand, 2:1 Resonance in the delayed nonlinear mathieu equation, Nolinear Dynamics, in press.

[5] H. Hu, E.H. Dowell, L.N. Virgin, Resonance of a harmonically forced Duffing oscillator with time delay state feedback. Nonlinear Dynamics 15 (1998) 311-327.

[6] Nayfeh AH, Mook DT. Nonlinear Oscillations, Wiley, New York, 1979.

[7] R. Bourkha, M. Belhaq, Effect of fast harmonic excitation on a self-excited motion in van der Pol oscillator, Chaos, Solitons & Fractals, in press.